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NOTES

ON SOME PROBLEMS INVOLVING RANDOM NUMBER OF RANDOM VARIABLES[1]

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1. Introduction. Frequently in various investigations, considerations on stochastic models involve sequences of random number of random variables. For instance, in many applications the number of experiments or observations in some interval of time $(0, t]$ is a chance variable. Of particular interest, for example, is the question of extreme values and sum of these observations in $(0, t]$.

Let $\tau(v)$ and ξ_v denote the time of vth observation and result of the observation, respectively, and suppose that for all $v = 1, 2, \cdots, 0 < \tau(v) < \tau(v+1)$, $\tau(v) \to \infty$ if $v \to \infty$ and $\xi_v > 0$. Assuming that the number of points $\tau(v)$ in $(0, t]$ is a chance variable, then $\tau(v)$ are random variables (rv's) as well. In addition it is supposed that $\tau(v)$ are continuous rv's.

In the following, attention is restricted to the next four functionals:

$$\inf_{\tau(v) \leqq t} \xi_v, \qquad \sup_{\tau(v) \leqq t} \xi_v \tag{1.1}$$

$$X(t) = \sum_{\tau(v) \leqq t} \xi_v, \qquad T(x) = \inf\{t; X(t) > x\}. \tag{1.2}$$

An attempt is made to determine a reasonable description of the extremes (1.1). In addition, the mathematical expectations and one-dimensional distribution functions (df's) of the processes (1.2) are determined.

2. Notations and definitions. Consider the probability space $(\Omega, \mathscr{A}, P)$. By definition $\mathscr{A}$ is the smallest σ-field which contains all subsets of Ω of the form $\{\tau(v) \leqq t\}$ and $\{X_v \leqq x\}$, where $X_v = \sum_{k=1}^{v} \xi_k$. It is supposed that X_v are continuous rv's such that with probability one, $X_v \to \infty$ if $v \to \infty$.

Let E_v^t and G_v^x be defined as follows:

$$E_v^t = \{\tau(v) \leqq t < \tau(v+1)\}, \qquad G_v^x = \{X_v \leqq x < X_{v+1}\} \tag{2.1}$$

then for all $i \neq j = 0, 1, \cdots, E_i^t \cap E_j^t = \theta, G_i^x \cap G_j^x = \theta, \bigcup_{v=0}^{\infty} E_v^t = \bigcup_{v=0}^{\infty} G_v^x = \Omega$, where θ denotes the empty set. By virtue of (2.1) it follows that:

$$P(E_v^t) = P\{\tau(v) \leqq t\} - P\{\tau(v+1) \leqq t\}. \tag{2.2}$$

$$P(G_v^x) = P\{X_v \leqq x\} - P\{X_{v+1} \leqq x\}. \tag{2.3}$$

Since $\{\tau(v)\}$ and $\{X_v\}$ are strictly increasing sequences of continuous rv's it follows

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