The Annals of Mathematical Statistics
1970, Vol. 41, No. 4, 1369–1372

ON A CLASS OF UNIFORMLY ADMISSIBLE ESTIMATORS OF A FINITE POPULATION TOTAL[1]

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1. Introduction. In a recent paper Godambe (1969) established the uniform admissibility of a class of estimators of a finite population total. In the present note we extend this class. The notation and definitions of this section follow that of Godambe (1969).

Any subset s of the integers, $1, \cdots, N$, which label the N distinguishable population units, is called a sample. A sampling design is defined by a probability mass function, p, on S, the set of all possible samples. Let x_i be the real (unknown) value associated with the ith population unit and let $\mathbf{x} = (x_1, \cdots, x_N) \in R^N$. Any real-valued function $e(\mathbf{x}, s)$ which depends on $\mathbf{x}$ only through those values x_i for $i \in s$ will be termed an estimator. We will be concerned with estimation of the population total, $T(\mathbf{x}) = \sum_1^N x_i$, under quadratic losses.

DEFINITION 1.1. For any given sampling design p, an estimator e' is said to *dominate* the estimator e if for all $\mathbf{x} \in R^N$

$$\textstyle\sum_S p(s)[e'(s, \mathbf{x}) - T(\mathbf{x})]^2 \leqq \sum_S p(s)[e(s, \mathbf{x}) - T(\mathbf{x})]^2$$

with strict inequality for at least one $\mathbf{x}$.

DEFINITION 1.2. A pair (e', p') of an estimator e' and a sampling design p' is said to *uniformly dominate* another pair (e, p) if for all $\mathbf{x} \in R^N$

$$\textstyle\sum_S p'(s)[e'(s, \mathbf{x}) - T(\mathbf{x})]^2 \leqq \sum_S p(s)[e(s, \mathbf{x}) - T(\mathbf{x})]^2$$

with strict inequality for at least one $\mathbf{x}$.

The notions of *admissibility* of an estimator for a given sampling design and that of *uniform admissibility* of a pair (e, p) for p in a class, C, of designs are then defined in the standard manner.

If $C_n = \{p \mid \sum_S p(s) n(s) = n\}$ where $n(s)$ is the cardinality of s then the main result of Godambe (1969) is that with respect to the class C_n the pair (e^*, p^*) is uniformly admissible where $e^*(s, \mathbf{x}) = \sum_{i \in s} x_i + \sum_{i \notin s} \lambda_i$, ($\lambda_1, \cdots, \lambda_N$ being *any* fixed values) and where p^* is *any* member of C_n.

DEFINITION 1.3. For $0 < n < N$, $D_n \equiv \{p \mid p(s) = 0 \text{ if } n(s) \neq n\}$ i.e., D_n is the class of fixed size sample designs.

The main result of this note is then

Received July 25, 1969; revised December 19, 1969.

[1] This research was partially sponsored by the National Science Foundation under Grant NSF-GP-8881.