The Annals of Mathematical Statistics
1970, Vol. 41, No. 5, 1604–1623

MARKOV RENEWAL PROCESSES WITH AUXILIARY PATHS

BY MANFRED SCHÄL

University of Hamburg

1. Introduction. A question of general interest in the theory of probability is that concerning the asymptotic behavior of a stochastic process. The purpose of the present paper is to investigate the ergodic properties of a class of stochastic processes characterized by the fact that the Markov property holds at an increasing sequence of stopping times $\{S_n\}$ called regeneration points.

In [9], [10] D. G. Kendall developed the analysis of such stochastic processes, which frequently occur in the theory of queues, by the method of the embedded Markov chain (MC). Replacing the continuous time parameter by the discrete parameter of an MC, he obtains results about the ergodic properties of these processes. In general, however, the limiting distribution of the embedded MC is not the limiting distribution of the original process. Therefore, the question of the relationship between these limiting distributions arises. After being solved in some special cases, this problem is now examined from a general point of view. An application of the present theory to the theory of queues will be given in a separate paper [15], where the system $M/G/1$ with state-dependent service times and the system $GI/M/1$ with state-dependent input will be dealt with.

A simple example for a stochastic process with regeneration points is a semi-Markov process (SMP) including as special cases Markov chains in discrete and continuous time. It is a fundamental structure of the class of stochastic processes studied in this paper that they are associated with an SMP. Therefore, results for SMP's by Pyke and Schaufele ([12]) turn out to be very useful.

By the same authors the concept of a Markov renewal process with auxiliary paths (MRPAP) was introduced, which is likely to include all stochastic processes with regeneration points arising from any practical situation. The name MRPAP is also used for the class of stochastic processes $(X_t, t \geqq 0)$ studied in the present paper although a slightly different definition is given. Pyke and Schaufele allow the sequence of regeneration points to have several accumulation points. This case will be excluded. On the other hand, for the most part of this paper we only require a weakened Markov property: If the value of the embedded SMP at a regeneration point is known, a statement of the past history of the SMP loses all its predictive value for the subsequent development of the MRPAP. However, for some purposes this class of processes is too large and a stronger Markov property is needed, which insures that the whole history of the MRPAP up to time S_n becomes irrelevant to its future development. MRPAP's enjoying this property are called MRPAP's in the strict sense.

2. Summary. Section 3 contains the definition of an MRPAP and some important remarks. In Section 4, a system of shift operators $\{\theta_n\}$ is introduced characterized

Received May 6, 1969.