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ADMISSIBILITY OF INVARIANT CONFIDENCE PROCEDURES FOR ESTIMATING A LOCATION PARAMETER

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1. Introduction. Let X be a random variable with a probability density $f(X-\theta)$ involving the location parameter θ. Let $\mathbf{x} = \{x_1, x_2, \cdots, x_n\}$ be n independent observations of X and let $g(\theta \mid \mathbf{x})$ be the conditional probability density of θ given $\mathbf{x}$ defined as follows:

$$g(\theta \mid \mathbf{x}) = \prod_{r=1}^{n} f(x_r - \theta) \cdot [\int_{-\infty}^{\infty} \prod_{i=1}^{n} f(x_i - \theta)\, d\theta]^{-1} \tag{1}$$

Let C_0 be the confidence procedure which assigns to the observed values $\mathbf{x}$, the confidence set for θ, given by

$$C_0(\mathbf{x}, \cdot) = \{\theta : g(\theta \mid \mathbf{x}) \geqq b\} \tag{2}$$

where $b > 0$ is some fixed constant. The procedure C_0 is translation invariant, i.e., if $x_i' = x_i + k$, $i = 1, 2, \cdots, n$, then the confidence set $C_0(\mathbf{x}', \cdot)$ is obtained by translating each point θ of $C_0(\mathbf{x}, \cdot)$ to $\theta + k$. It is easily verified that the expected Lebesgue measure of the confidence sets of C_0, viz. $E_\theta v C_0(\mathbf{x}, \cdot)$ is equal to some constant v_0 for all θ. Similarly the inclusion probability, i.e., the probability that the "true value" θ is included in the observed confidence set $C_0(\mathbf{x}, \cdot)$ is independent of θ and equal to $(1-\alpha)$ say. Further C_0 has the minimax property that amongst the confidence procedures C with given lower confidence level $(1-\alpha)$, C_0 minimizes the maximum expected Lebesgue measure of the confidence sets viz. $E_\theta v C(\mathbf{x}, \cdot)$. This minimax property has been proved by Kudō (1955) and is also deducible from results proved by Valand (1968).

In the following we investigate the question whether the procedure C_0 is unique in having the minimax property and show that subject to the density $f(x)$ satisfying two conditions, the procedure is essentially unique, i.e. to say, it is unique if we treat as equivalent procedures whose confidence sets for almost all $\mathbf{x}$, differ from each other at most by null subsets of the parameter space. The uniqueness is proved in the extended class of randomized confidence procedures.

Investigating a conjecture of Stein (1958) a similar uniqueness property of the usual confidence sets for univariate and bivariate normal populations was proved previously (1969). The present result contains the previous result for the univariate normal population as a particular case.

2. Preliminaries. X is a random variable with a probability density $f(x-\theta)$ where θ is a location parameter; $x_1, x_2, \cdots, x_n$ denote n independent observations of X: $\mathbf{x} = (x_1, x_2, \cdots, x_n)$ denotes a point in the n-dimensional Euclidean sample space R; θ assumes values in the parameter space $\Omega = (-\infty, \infty)$; on R, Ω and the

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