The Annals of Mathematical Statistics
1970, Vol. 41, No. 5, 1510–1529

AN INFINITESIMAL DECOMPOSITION FOR A CLASS OF MARKOV PROCESSES[1]

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1. Introduction. An unavoidable initial issue in treating many questions on Markov processes is that specifying the objects to be studied. There are various different possible means of specification, and the one chosen will depend, of course, both upon the purposes of the work at hand and upon the particular process, or class thereof, to be specified. If we assume the process to have stationary transition probabilities, one important means (which provides a good analytical access to the process) is to give an "infinitesimal generator" defined in any one of several ways. A standard example is based on the well-known formula of Lévy and Khintchine for the characteristic functions of infinitely divisible processes in R^n, which can be interpreted as giving an instantaneous decomposition of the process into independent Gaussian and Poissonian components ([6] page 550).

Quite a different method of specification is usually employed in studies of general potential-theoretic questions. Here the necessary requirements involve stopping times, and are therefore expressed qualitatively in terms of the behavior of the path functions of the process together with some general requirements on the regularity of the transition function.

This distinction in method naturally raises the problem of determining how the two approaches are connected, which is not an easy matter to settle. Much recent work on Markov processes, particularly that of A. V. Skorokhod [12]–[15], can be considered as being directed toward obtaining the explicit generators of a sufficiently wide abstract class of processes.[2] Partly for technical reasons, this work is often done in the reverse order. The generators are specified first and the corresponding processes are then constructed. Up to the present, however, the constructive method has not attained the goal of producing a complete class of processes free of unnatural restrictions, except perhaps in the very particular case of one-dimensional diffusion processes. On the other hand, general methods involving square integrable martingales have been developed recently for treating the same problem by starting with the processes themselves. These methods, which are due to several French, Russian, and Japanese probabilists, are fundamental to the present paper.[3]

Received March 25, 1969.

[1] This paper was written while the author was a Sloan postdoctoral fellow.

[2] Under the term "explicit generator" we do not include the general existence theories of the strong, weak or Dynkin generators, but only those generators in which the Gaussian and Poissonian components are distinguished.

[3] The related problem of establishing properties such as quasi-left continuity and the strong Markov property for processes whose generator or semigroup is known has also been extensively studied, but it seems to be quite separate from that of obtaining generators for a given class of