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AN APPLICATION OF EXTREME VALUE THEORY TO RELIABILITY THEORY[1]

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0. Introduction. The limiting distribution of the maximum term in a sequence of independent, identically distributed random variables was completely analysed in a series of works by many writers, culminating in the comprehensive work of Gnedenko [4]. Results for order statistics of fixed and increasing rank were obtained by Smirnov [10], who completely characterized the limiting types and their domains of attraction. Generalizations of these results for the maximum term have been made by several writers; Juncosa [7] dropped the assumption of a common distribution, Watson [11] proved that under slight restrictions the limiting distribution of the maximum term in a stationary sequence of m-dependent random variables is the same as in the independent case, and Berman [1] studied exchangeable random variables and samples of random size. A bibliography and discussion of applications is contained in the book by Gumbel [6].

This paper extends the classical theory by introducing a model from reliability theory—essentially a series system with replaceable components. It is shown that the asymptotic distribution of system lifetime can belong to one of two types when the number of spares is fixed or of a smaller order than the total number n of components, as n becomes infinite, and that these limiting distributions are the same as those obtained by Gnedenko, Chibisov [2] and Smirnov.

1. Notation and classical results. Throughout this paper, the distribution function of a random variable X will be denoted by $P\{X \leqq x\} = F(x)$, and the tail of the distribution by $P\{X > x\} = \bar{F}(x)$. The abbreviation "df" will be used for distribution function. A df will be called *proper* if:

$$\lim_{x\to\infty} F(x) = 1, \qquad \lim_{x\to-\infty} F(x) = 0$$

and not all its mass is concentrated at one point. Two df's $F_1(x)$ and $F_2(x)$ are said to be of the *same type* if there exist constants $A > 0$ and B such that: $F_1(Ax+B) = F_2(x)$ for all values of x. Unless otherwise stated, all df's will be assumed proper and all limiting df's should be taken to mean limiting types of df's. Let $X_1, X_2, \cdots, X_n, \cdots$ be a sequence of independent random variables with common distribution $F(x)$, and let $\xi_n = \min(X_1, X_2, \cdots, X_n)$. Then the limiting df of ξ_n belongs to exactly one of three types [4]; that is to say, if there exist sequences of normalizing constants $\{a_n > 0\}$ and $(b_n\}$ and a df $G(x)$ such that:

$$\lim_{n\to\infty} P\{a_n^{-1}(\xi_n - b_n) \leqq x\} = G(x)$$

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