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ABSTRACT OF PAPERS

(Abstracts of Papers contributed by title)

70T-88. On pairing random observations from a bivariate distribution. MILTON C. CHEW, JR., Rensselaer Polytechnic Institute.

Suppose a random sample of size n is taken from a known bivariate distribution, whose density $f(x,y)$ possesses a "monotone likelihood ratio," i.e. for all $x_1 < x_2$ and $y_1 < y_2$, one of the following inequalities always holds: $f(x_1,y_1)f(x_2,y_2) \lesssim f(x_1,y_2)f(x_2,y_1)$. Included in such a class are the normal and trinomial distributions. When the rank order of the x and y data is known, but their correct pairwise ordering is not, it is desirable to optimally "reconstruct" the original bivariate sample. Such a situation occurs when one wishes to "decode" the received y-data into the x-data transmitted over a memoryless channel. It is easily seen that the Maximum Likelihood Pairing (MLP) maximizes the probability of a perfect match, or "reconstruction." A more difficult objective is to maximize the expected number of correct (x,y) pairs. In this case it is unlikely (though unproved) that the optimal pairing depends only on the rank order of the data, and not their values. It is shown, however, that the MLP is better than an arbitrary pairing, whose expected number of correct pairs is unity for all n. The problem is treated as one of combinatorics, and use is made of the rencontres numbers used in the random matching problem. (Received July 23, 1970.)

70T-89. Nonexistence of a single-sample selection procedure whose $P(CS)$ is independent of the variances. EDWARD J. DUDEWICZ, The University of Rochester.

Suppose an experimenter has k populations $\pi_1, \cdots, \pi_k$, where observations from π_i are normally distributed $N(\mu_i, \sigma_i^2)$ $(1 \leqq i \leqq k)$ with $\mu_1, \cdots, \mu_k$, $\sigma_1^2, \cdots, \sigma_k^2$ all unknown. Further, suppose the goal is to select that population which has the largest mean, say $\mu_{[k]}$ (where $\mu_{[1]} \leqq \cdots \leqq \mu_{[k]}$ denote the ordered means). Bechhofer [*Ann. Math. Statist.* **25** (1954) 16–39] discussed this problem when $\sigma_1^2 = \cdots = \sigma_k^2 = \sigma^2$ with σ^2 known, Bechhofer, Dunnett and Sobel [*Biometrika* **41** (1954) 170–176] gave a two-sample solution for the case $\sigma_1^2 = \cdots = \sigma_k^2 = \sigma^2$ with σ^2 unknown, and various authors considered sequential solutions for these two cases. No satisfactory solution is available for the problem with $\sigma_1^2, \cdots, \sigma_k^2$ unknown. One reason is that certain types of single-sample procedures do not exist to provide such a solution. This result, stated but not proved by Bechhofer, Dunnett and Sobel [*loc. cit.*], is given a rigorous treatment, using results of Dantzig [*Ann. Math. Statist.* **11** (1940) 186–192], Stein [*Ann. Math. Statist.* **16** (1945), 243–258], and Hall [*Ann. Math. Statist.* **30** (1959), 964–969]. (Received July 24, 1970.)

70T-90. Confidence intervals for power, with special reference to medical trials. EDWARD J. DUDEWICZ, The University of Rochester.

Suppose that one has observed $X_1, \cdots, X_n$, independent random variables, each normally distributed $N(\delta, \sigma^2)$ where δ $(-\infty < \delta < +\infty)$ and σ^2 $(0 < \sigma^2)$ are both unknown. For the problem of testing H: $\delta = 0$ it is customary to use a t-test (at level $\alpha = 0.05$, e.g.); one then calculates $t = \bar{X}n^{\frac{1}{2}}/\hat{\sigma}$ (where $\hat{\sigma}^2 = \sum(X_i - \bar{X})^2/(n-1)$) and compares it with a critical value $t_\alpha(n)$. This yields a Type I error of α, and a power which is a function of $d = \delta n^{\frac{1}{2}}/\sigma$. Since one does not know σ, one cannot answer the question "What is the probability that I would have detected a difference $|\delta| \geqq 0.2$ (e.g.)?" A common recommendation, usually implicit [*Biometrika Tables for Statisticians* **1**, 3rd ed. (1966) 25] but sometimes explicit [*Experimental Design: Procedures for the Behavioral Sciences*. R. E. Kirk (1968) 108] is essentially that one act as if $d = \delta n^{\frac{1}{2}}/\hat{\sigma}$ in such power evaluations. We note that if one takes a confidence interval on σ^2 with confidence coefficient γ (e.g. $0.90 \leqq \gamma < 1$), one can obtain a confidence interval for the power at δ. This is compared with the usual power estimate, and the latter is found to be misleadingly large. An application illustrating the technique (on data from a study of the effect of digoxin in acute myocardial infarction) is given. Such power analysis applies to F-tests, etc. (Received July 24, 1970.)