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A COMPLETE CLASS THEOREM FOR MULTIDIMENSIONAL ONE-SIDED ALTERNATIVES

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1. Introduction. For testing a simple hypothesis in an exponential family, Birnbaum (1955) showed that the class of tests with closed convex acceptance regions, say D_0, forms an essentially complete class of decision functions. The original proof of this result was incomplete and a complete proof appeared in Matthes and Truax (1967) (hereafter referred to as M–T). In discussing the minimal completeness of the class D_0, Birnbaum considered testing $(\psi_1, \psi_2) = (0, 0)$ against $\{(\psi_1, \psi_2) \mid \psi_1 \geqq 0,\ \psi_2 \geqq 0,\ \psi_1 + \psi_2 > 0\}$ where $(\psi_1, \psi_2)'$ is the two-dimensional parameter vector of an exponential family. For this problem, Birnbaum showed that D_0 is not essentially minimal complete and he characterized a subset of D_0 which is essentially minimal complete under certain conditions. In this paper we present a generalization of this two-dimensional one-sided result by Birnbaum.

In recent years, a number of authors have considered the problem of testing that the mean of a multivariate normal distribution is 0 against the alternative that the mean is in a closed convex cone (in particular, the positive orthant). For example, Nuesch (1966) and Perlman (1969) were concerned with the derivation of, and distribution theory for, the likelihood ratio test for such a problem. A related problem is that of testing the equality of components of a mean vector versus an ordered alternative. Bartholomew (1959a,b, 1961a, 1961b) and Kudô (1963) have discussed this problem in detail. Also, Oosterhoff and Van Zwet (1967) considered Birnbaum's original one-sided problem in two dimensions in their paper on the combination of independent test statistics.

In Section 2 of this paper, we present some results concerning convex sets and convex cones. These results are used in Section 3 to establish a complete class theorem for testing a simple hypothesis against certain one-sided alternatives when the underlying distribution is an exponential family. In Section 4, this result is extended to include the case of nuisance parameters.

It is assumed that the reader is familiar with the results and methods in M–T. Certain proofs in this paper are rather abbreviated as the arguments parallel those in M–T.

2. Preliminary results. Let Φ be the class of all non-empty closed convex sets in R^k-k-dimensional Euclidean space. If V is a closed convex cone, $V \neq R^k$ and

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