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BEHAVIOR OF MOMENTS OF ROW SUMS OF ELEMENTARY SYSTEMS

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1. Introduction. Bawly [1] studied the convergence of a sequence of distribution functions of row sums of an elementary system, i.e. (Gnedenko [3] page 316) row sums of uniformly small random variables which converge in distribution to an infinitely divisible law with bounded variance. He introduced their so-called *accompanying laws* ([4] page 98, see Section 2), showed them to be asymptotic to the row sum distributions, and thus obtained necessary and sufficient conditions for the convergence in law of the row sums.

In this paper, a study is made of the behavior of the moments of the sequence of row sums, using the accompanying laws, and *their* moments. First, the cumulants of the sequence of row sums are shown to be closely related to those of the accompanying laws. This leads to necessary and sufficient conditions for the convergence of the moments of row sums to those of the limit distribution. These conditions include, as a special case, the Lindeberg conditions of even integer order which are necessary and sufficient for the convergence of moments in the central limit theorem (see [2]). The results of Section 1 of [2] are therefore placed in a natural and more general setting.

Section 4 contains a brief survey of the main results of the paper.

2. Preliminaries. Let $X_{n1}, X_{n2}, \cdots, X_{nj_n}$ be independent random variables for each $n = 1, 2, \cdots$, with $EX_{nj} = 0$ and $DX_{nj} < \infty$ for $j = 1, 2, \cdots, j_n$, $n = 1, 2, \cdots$; where DX denotes the variance of the random variable X. Assume that the $\{X_{nj}\}$ form an *elementary system* (see Gnedenko, page 316), i.e. that

$$\max_{j \leqq j_n} DX_{nj} \to 0 \qquad \text{as} \quad n \to \infty \tag{1}$$

and

$$\sum_j DX_{nj} \leqq \text{some } C < \infty \tag{2}$$

for all $n = 1, 2, \cdots$.

Let $F_{nj}(\cdot)$ denote the distribution function and $f_{nj}(\cdot)$ the characteristic function of X_{nj}. Also, for each $n = 1, 2, \cdots$, let $Y_{n1}, \cdots, Y_{nj_n}$ be independent random variables whose characteristic functions are given by $\phi_{nj}(t) = E \exp(itY_{nj}) = \exp(f_{nj}(t) - 1)$, and write

$$S_n = \sum_j X_{nj}, \qquad \text{and}$$

$$T_n = \sum_j Y_{nj}.$$

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