

EXTRAPOLATION AND INTERPOLATION OF STATIONARY GAUSSIAN PROCESSES¹

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1. Introduction. The problem of extrapolation of a stationary Gaussian process x using the whole past $x(t): t \leq 0$ was solved about 1940 by A. N. Kolmogorov [15], M. G. Krein [16], and N. Wiener [27]. The purpose of this paper is to present the mathematical tools needed to solve the extrapolation problem if only part of the past is available. M. G. Krein [20] did this by applying the solution of the inverse spectral problem as initiated by Gel'fand-Levitan [10] and perfected by himself [17-19]. Dym-McKean [8] used a second method involving spaces of integral (entire) functions, of the kind introduced and extensively studied by L. de Branges [5, 6].

Both methods take advantage of the fact that the study of the short process $x(t): |t| \leq T +$ is isomorphic, via the map $x(t) \rightarrow \exp(i\gamma t)$, to the study of the class of

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