

ADMISSIBILITY OF ESTIMATORS OF THE MEAN VECTOR OF A MULTIVARIATE NORMAL DISTRIBUTION WITH QUADRATIC LOSS¹

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