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USE OF TRUNCATED ESTIMATOR OF VARIANCE RATIO IN RECOVERY OF INTER-BLOCK INFORMATION[1]

By K. R. Shah

University of Waterloo and Cornell University

1. Introduction. As is well known, a preliminary step in the recovery of inter-block information is to estimate the ratio of inter-to intra-block variances. Under the infinite models generally used in the literature, the true value of this ratio exceeds unity. However, when this ratio is estimated using the data, the estimate may turn out to be less than unity. In such cases it is usually recommended that the value of this estimate should be taken as unity. As pointed out by Yates (1939) this amounts to estimating the treatment effects as in a completely randomised design.

Many authors have considered combined inter-and intra-block estimators of treatment differences based on an untruncated estimator of the variance ratio. However, it is usually felt that it would be better to use a truncated estimator of variance ratio. (Stein (1966) has conjectured this.) In this paper it is shown that in *any* incomplete block design for a class of estimators of variance ratio (which includes the ones considered by the above authors) truncation at any point less than the true value leads to a smaller variance for a combined estimator of a treatment difference. (If previous experience with the experimental material indicates that unity is not a safe lower bound one might consider truncation at zero.)

A table is presented to demonstrate that much of the gain due to recovery of inter-block information could be lost by using an untruncated estimator of the variance ratio.

2. Main results. Results of this section can be readily given using the notation used in Shah (1964) and hence we shall use this notation without introducing it explicitly. In Shah (1964), the variance of $\bar{t}_s(\rho^*)$, the combined estimate of τ_s, the the sth canonical treatment contrast using ρ^* as an estimate of ρ, the variance ratio was expressed as

$$V(\bar{t}_s(\rho^*)) = V(\bar{t}_s(\rho)) + [c_s{}^2/a_{0s}^2(1+\rho c_s)^2]E(\omega_s{}^2) \tag{2.1}$$

where $\bar{t}_s(\rho)$ is the estimate of τ_s based on true value of ρ and ω_s is defined as $\omega_s = [(\rho^* - \rho)/(1+\rho^* c_s)]z_s$. [$c_s$, a_{0s} and z_s are all defined in Shah (1964). z_s is in fact proportional to the difference between intra-and inter-block estimates of τ_s, while c_s and a_{0s} are constants depending upon design parameters.] It was shown in Roy and Shah (1962) that (2.1) holds provided that

$$E\{\omega_s(\rho^*) = 0 \quad \text{and} \quad V\{\omega_s(\rho^*)\} < \infty. \tag{2.2}$$

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