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LOCAL THEOREMS IN STRENGTHENED FORM FOR LATTICE RANDOM VARIABLES

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1. Introduction. Let $\{X_n\}$ be a sequence of independent integral-valued lattice random variables such that the distribution of X_n is one of the distinct nondegenerate distributions $H_1, \cdots, H_r$ $(r \geqq 2)$. With the assumption that there are sequences $\{A_n\}$ and $\{B_n\}$ $(0 < B_n \to \infty)$ such that $Z_n \equiv B_n^{-1}(X_1 + \cdots + X_n) - A_n$ converges in law to a nondegenerate distribution G, this paper investigates some conditions which are sufficient for $\{X_n\}$ to satisfy a local theorem in strengthened form.

2. Discussions and results. V. M. Kruglov [2] noted that a result of A. A. Zinger [5] implies that G has an absolutely integrable characteristic function ϕ and, therefore, a bounded density g.

We say that a local limit theorem holds for $\{X_n\}$ if

$$\lim_{n\to\infty} \sup_{N\in Z} \left|B_n P(\textstyle\sum_{i=1}^n X_i = N) - g((N/B_n) - A_n)\right| = 0,$$

where Z denotes the set of all integers.

We say that $\{X_n\}$ satisfies a local theorem in strengthened form (L.T.S.) if a local limit theorem (in the usual form) holds for any sequence $\{X_n'\}$ which differs from $\{X_n\}$ only by a finite number of terms.

Since it is possible that for some i among $1, \cdots, r$ the number of times that H_i appears among the distributions of $X_1, \cdots, X_n$ eventually does not depend on n, we let $F_1, \cdots, F_k$ be those among $H_1, \cdots, H_r$ for which this does not occur. Let $n_i(n)$ denote the number of times that F_i appears among the distributions of $X_1, \cdots, X_n$. Then $n_i(n) \to \infty$ as $n \to \infty$ for $i = 1, \cdots, k$. Let h_i denote the maximum span of F_i for $i = 1, \cdots, k$.

LEMMA (Petrov). *A necessary condition that* $\{X_n\}$ *satisfies* L.T.S. *is that*

$$\text{g.c.d.}(h_1, \cdots, h_k) = 1. \tag{1}$$

(Here, as usual, g.c.d. means the greatest common divisor.)

The proof of this Lemma is contained in the proof of Theorem 1 of [4].

Because of this lemma, we will always assume that (1) holds.

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