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CORRECTION NOTE

CORRECTIONS TO "STRONG CONSISTENCY OF CERTAIN SEQUENTIAL ESTIMATORS"

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In the above paper (*Ann. Math. Statist.* **40** 1492–1495), the main consistency result (Theorem 3.4) uses Theorem 2.7 to justify the assertion that $\mathscr{C}^* = \downarrow \lim_i \mathscr{C}_{t_i} \equiv \{\varnothing, \Omega\}$. However, Theorem 2.7 as stated is incorrect: $\mathscr{C}^* = \downarrow \lim \mathscr{C}_{N_i}$ can properly contain $\mathscr{C}_{N_\infty}$. Proposition 1 below illustrates an instance of this. To get around this difficulty, Proposition 2 below gives a sufficient condition that $\mathscr{C}^* \equiv \{\varnothing, \Omega\}$. The condition is seen to hold for a large variety of examples, including those considered in the paper.

PROPOSITION 1. *Let N be a random index and let $N_n = \max\{N, n\}$, $n = 1, 2, \cdots$. Then for any decreasing sequence $\{\mathscr{C}_i\}$ with $\mathscr{C}_\infty = \downarrow \lim_i \mathscr{C}_i$, $\{N_n: 1 \leqq n \leqq \infty\}$ is C-ordered and $\mathscr{C}_{N_n} \downarrow \mathscr{B}(\mathscr{C}_\infty, (N < \infty))$, the σ-field generated by $\mathscr{C}_\infty$ and the set $(N < \infty)$.*

PROOF. If $C \in \mathscr{C}_{N_n}$,

$$C = \sum_{k=1}^{\infty} C_{kn}(N_n = k) \cup C_{\infty n}(N_n = \infty) \tag{1}$$

$$= C_n(N \leqq n) \cup \sum_{n+1}^{\infty} C_{kn}(N = k) \cup C_\infty(N = \infty),$$

where $C_{kn} \varepsilon \mathscr{C}_k$, $1 \leqq k \leqq \infty$ and we write $C_{nn} = C_n$ and $C_{\infty n} = C_\infty$ (note that the latter set does not depend on n). Clearly any set in $\mathscr{C}_{N_{n+1}}$ is of this form (with $C_n = C_{n,n+1} = C_{n+1}$), so $\mathscr{C}_{N_{n+1}} \subset \mathscr{C}_{N_n}$. Thus $\{N_n: 1 \leqq n \leqq \infty\}$ is C-ordered. Let $C^* = \downarrow \lim_n \mathscr{C}_{N_n}$. We note that for all n, $\mathscr{C}_\infty \subset \mathscr{C}_{N_n}$ and $(N < \infty) = (N_n < \infty) \in \mathscr{C}_{N_n}$. Hence $\mathscr{C}^* \supset \mathscr{B}(\mathscr{C}_\infty, (N < \infty))$. This already contradicts Theorem 2.7, which asserts in this case that $\mathscr{C}^* = \mathscr{C}_\infty$.

To establish the reverse inclusion for $\mathscr{C}^*$, choose $C \in \mathscr{C}^*$. Then for all n, C has a representation as in (1). Fix m. For $n > m$, it follows from (1) that $C(N \leqq m) = C_m(N \leqq m) = C_n(N \leqq m)$. Thus $\lim 1_{C_n} = 1$ on $(N \leqq m)$. Let $C_{\infty *} = \limsup C_n \in \mathscr{C}_\infty$. Then $C(N \leqq m) = C_{\infty *}(N \leqq m)$. Letting $m \to \infty$ then shows that $C = C_{\infty *}(N < \infty) \cup C_\infty (N = \infty)$. Thus $\mathscr{C}^* \subset \mathscr{B}(\mathscr{C}_\infty, (N < \infty))$. □

We note that if N is a stopping time in Proposition 1, then so are the N_n. Thus Theorem 2.7 is not even true in general for C-ordered stopping times. If one adds the hypothesis $N_\infty < \infty$ with probability one, Theorem 2.7 is true and the proof given is valid. (Whether the theorem remains true under the weaker hypothesis: for all i, $N_i < \infty$ with probability one, is not known. Note that in Proposition 1, $N_n < \infty$ with probability one if and only if $N < \infty$ with probability one and then $\mathscr{C}^* \equiv \mathscr{C}_\infty$.) Of course the case of primary interest in the paper is $N_\infty \equiv \infty$, so some suitable alternative to Theorem 2.7 seems necessary.