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ADMISSIBLE ESTIMATORS, RECURRENT DIFFUSIONS, AND INSOLUBLE BOUNDARY VALUE PROBLEMS[1]

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CONTENTS

1. Introduction.

1.1. *Summary*. Consider the problem of estimating the mean of a multivariate normal distribution on the basis of one observation (or more) from that distribution. Take squared error as the loss function—the mathematically simplest choice, and a frequently studied one. We are interested in determining necessary and sufficient conditions for an estimator, δ, to be admissible.

C. Stein (1956) proved that the best invariant estimator ($\delta(x) = x$) is admissible if m—the dimension of the multivariate normal distribution—satisfies $m \leqq 2$ and is inadmissible if $m \geqq 3$. He also gave a heuristic argument which pleads the case that for sufficiently large m the best invariant estimator must be inadmissible. But this heuristic argument gives no indication of the fact that "sufficiently large" m is really $m = 3$.

There is another interesting division between dimensions $m = 2$ and $m = 3$ with which probabilists and statisticians are familiar. Brownian motion is recurrent in dimensions $m = 1, 2$ and is transient if $m \geqq 3$. A variant of the heuristic argument mentioned above pleads the case that for sufficiently large dimension Brownian motion must be transient, but again there is no indication that $m = 3$ is "sufficiently large."

We have been able to determine a necessary and sufficient condition for an estimator having bounded risk to be admissible. We are also able to extend our considerations to many estimators having unbounded risk.

In the process of establishing this condition we develop a close *mathematical* connection between the statistical question of admissibility and the probabilistic question of recurrence. This connection goes far beyond the invariant cases men-

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