The Annals of Mathematical Statistics
1971, Vol. 42, No. 4, 1206–1240

NON-LOCAL ASYMPTOTIC OPTIMALITY OF APPROPRIATE LIKELIHOOD RATIO TESTS

BY L. D. BROWN

Cornell University

CONTENTS

1. Introduction. Suppose $x_1, x_2, \cdots, x_n$ are n independent identically distributed observations on a random variable having distribution F_θ, $\theta \in \Theta$. Suppose it is desired to test the null hypothesis $\theta \in \Theta_0$ versus the alternative $\theta \in \Theta_1 = \Theta - \Theta_0$. Several different methods have been proposed to relate asymptotic performance (as $n \to \infty$) of two (or more) different sequences of tests. It may be said that these methods fall into two broad categories.

First, there are "local" methods such as Pitman efficiency and its generalizations, see Noether (1955), Neyman (1959). In these approaches properties of tests are compared at appropriately chosen sequences of points in the alternative hypothesis (and perhaps also a sequence of points in the null hypothesis). A different alternative point is chosen for each sample size n, and properties are compared as $n \to \infty$. Generally the sequences of points are chosen so that the probabilities of type I error and type II error at the chosen points remain bounded away from zero for at least one of the sequences of tests under comparison. The characteristic of these methods which makes the name "local" appropriate is that the sequence of points in the alternative hypothesis gets arbitrarily close to the null hypothesis.

On the other hand there are the "non-local," or "fixed alternative," methods. In these methods the rate of exponential convergence to zero of the significance level and/or of type II error at a particular point are examined. Denoting probabilities of type I and type II error by α_n and β_n, respectively, one thus looks either at $\lim_{n\to\infty} n^{-1} \log \alpha_n(\theta)$ for fixed $\theta \in \Theta_0$, or more usually $\lim_{n\to\infty} n^{-1} \log \sup_{\theta\in\Theta_0} \alpha_n(\theta)$, and/or at $\lim_{n\to\infty} n^{-1} \log \beta_n(\theta)$ for fixed $\theta \in \Theta_1$. (See Section 2 for explicit definitions of these terms and others used in the introduction.)

While one might consider other measures of rate of approach of α_n and/or β_n to 0, the exponential measurement described above seems to be right for "non-local" asymptotic properties. It is discriminating enough to provide non-trivial com-

Received October 3, 1968; revised January 4, 1971.