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ON THE NUMBER OF GENERATORS OF SATURATED MAIN EFFECT FRACTIONAL REPLICATES[1]

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1. Introduction. Recently Paik and Federer (1970) have presented a method to generate from a given saturated main effect plan of the s^m factorial a class of plans such that the determinant of the information matrix remains invariant. They did not however enumerate the total number of generators, nor did they enumerate the number of plans generated by each generator. This paper solves this problem using a group-theoretic approach.

2. Some known results. Consider the well-known Abelian group G of order s^m, which consists of row vectors of the form $u' = (u_1, u_2, \cdots, u_m)$, where prime indicates transpose and each u_i is an element of $GF(s)$, s being a prime or a power of a prime. The following lemma is given in most elementary text books on group theory:

LEMMA 2.1. (a) *Every subgroup of G is of order s^k, k being an integer.*

(b) *The number of distinct subgroups of order s^k of G is equal to*:

$$\alpha = \alpha(k, m, s) = \frac{(s^m-1)(s^m-s)(s^m-s^2)\cdots(s^m-s^{k-1})}{(s^k-1)(s^k-s)\,(s^k-s^2)\cdots(s^k-s^{k-1})}. \tag{2.1}$$

(c) *The number of cosets into which G can be partitioned relative to a subgroup of order s^k is equal to*:

$$\beta = \beta(m, s, k) = s^{m-k}. \tag{2.2}$$

Now, let F denote the set of $[m(s-1)+1] \times m$-matrices such that the rows of each matrix form an $[m(s-1)+1]$-subset of G. Note that the order of F is equal to:

$$\gamma = \gamma(m, s) = \binom{s^m}{m(s-1)+1}. \tag{2.3}$$

Consider linear functionals f from G into R(= real line) of the form:

$$f(u') = c'\phi \tag{2.4}$$

where c' is an $[m(s-1)+1]$-row vector of known real coefficients and ϕ is an $[m(s-1)+1]$-column vector of unknown real parameters. These functionals lead to the fact that to each $A \in F$ there corresponds a square matrix B of order $[m(s-1)+1]$, the rows of which are vectors of the form c'. Denote the set of B's by H, which incidentally has the same order as F. The following theorem has been proved recently by Paik and Federer (1970):

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