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THE SERIAL CORRELATION COEFFICIENTS OF WAITING TIMES IN THE STATIONARY GI/M/1 QUEUE[1]

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1. Introduction. The serial correlation coefficients $\{r_n\}$ of a stationary sequence of waiting times in the GI/G/1 queueing system have recently been studied by Daley (1968a) and Blomqvist (1968, 1969). From a practical point of view, knowledge of the properties of $\{r_n\}$ is useful for obtaining the variance of the mean of a sample of waiting times, and thus for obtaining some idea of required sample sizes for estimation and simulation. For example, Blomqvist (1968) has defined, for a stable GI/G/1 system with zero initial waiting time, an estimator for the expected stationary waiting time which is based on a sample of successive waiting times. He shows that the mean square error of this estimator can be expressed in terms of Σr_n. Blomqvist (1969) has given heavy traffic approximations for Σr_n and r_n. The special case of the stationary M/G/1 queue has been treated in some detail by Daley (1968a) and Blomqvist (1967).

In this paper we consider the stationary GI/M/1 queue, thus complementing the work of Daley and Blomqvist, and also of Daley (1968b) and Pakes (1971) where a discussion is given of the serial correlation coefficients of a stationary sequence of queue lengths embedded at the epochs of arrival of successive customers. In Section 3 we evaluate $\{r_n\}$ for the stationary GI/M/1 queue and in Section 4 we discuss heavy traffic approximations.

A quantity related to waiting time is the sojourn, or waiting plus service, time of a customer. In Section 5 we consider $\{\tau_n\}$, the serial correlation coefficients of a stationary sequence of sojourn times in the GI/G/1 queue. Using the results and methods of Section 3, we evaluate $\{\tau_n\}$ for the stationary GI/M/1 queue and thus show the equality of this sequence and the sequence of correlation coefficients of a stationary sequence of queue lengths embedded at arrival epochs.

2. Notation. We consider a GI/G/1 queue where the nth arriving customer is denoted by $C_n(n = 0, 1, \cdots)$, T_n is the interarrival time of C_n and C_{n+1}, and S_n, W_n and $V_n = W_n + S_n$ are the service, waiting and sojourn times of C_n, respectively. For $n = 0, 1, \cdots$, we let $A(x) = \Pr\{T_n \leqq x\}$ and $B(x) = \Pr\{S_n \leqq x\}$ $(x \geqq 0)$ with $A(0+) = B(0+) = 0$. We assume that $\{S_n\}$ and $\{T_n\}$ are independent sequences of mutually independent random variables and we put $U_n = S_n - T_n$ with $U(x) = \Pr(U_n \leqq x)$ $(-\infty < x < \infty)$. We denote the moments of the inter-arrival times by $\lambda_r = E(T_n^r)$ and of the service times by $\mu_r = E(S_n^r)$ $(r = 1, 2, \cdots)$, when they exist. We always assume $\lambda_1, \mu_1 < \infty$.

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