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ON THE ADMISSIBLE ESTIMATORS FOR CERTAIN FIXED SAMPLE BINOMIAL PROBLEMS

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0. Introduction. Let X be a binomial random variable, $b(n, p)$, and f a continuous real valued function on [0, 1]. We consider the problem of estimating $f(p)$ with, for example, squared error loss. It is well known (Wald (1950), Le Cam (1955)) that the class of Bayes procedures is a complete class. Further, since any unique Bayes estimator is admissible, our concern with admissibility centers on the non-unique Bayes estimators. In Section 1, this single parameter problem is considered. In Section 2, $X_1, \cdots, X_q$ are assumed independent binomials, $b(n_i, p_i)$, $i = 1, \cdots, q$, and each of $f_i(p_i)$, $i = 1, \cdots, q$, or $f(p_1, \cdots, p_q)$ is to be estimated with summed squared error loss for the first problem and squared error loss for the second, and in each case with a continuity assumption on the f. In Section 3, the $X_1, \cdots, X_q$ are assumed to have a multinomial distribution and the analogues of the problems of Section 2 are considered. The main result of this note is that for these problems the classes of admissible estimators are closed in the topology of pointwise convergence of the estimators. Also, a procedure is given for constructing the non-unique Bayes estimators. A few examples, for which the admissibility is generally known, are included to illustrate the construction. For the problem of estimating each $f_i(p_i)$ when the X_i are independent, it is shown that there is no Stein effect (Stein (1956)). That is, $\delta = (\delta_1, \cdots, \delta_q)$ is admissible if δ_i is admissible for the problem of estimating $f_i(p_i)$ based only on X_i. Section 4 contains some more or less obvious extensions to related problems.

The method employed is contained in the following simple observation. Let P_α, $\alpha \in A$, be a family of discrete probability densities for X. Suppose our interest is in estimators, $\delta(X)$, of $f(\alpha)$ when the loss is, say, squared error. Let A_0 be a closed subset of A, $D^c = (x : P_\alpha(X = x) > 0$ for some $\alpha \in A_0)$, and D the remainder of the sample space, which we suppose is not empty. The risk of δ is

$$\rho(\alpha, \delta) = \sum_{x \in D^c}(\delta(x) - f(\alpha))^2 P_\alpha(X = x) + \sum_{x \in D}(\delta(x) - f(\alpha))^2 P_\alpha(X = x \mid D)P_\alpha(D).$$

Suppose the restriction of δ to D^c is admissible for the problem of estimating $f(\alpha)$ if α is restricted to A_0 and further that no other estimator for the restricted problem has the same risk. Then plainly the risk of δ can only be minorized by another estimator with the same determination on D^c. Also, δ is admissible if and only if its restriction to D is admissible for the problem of estimating $f(\alpha)$ if α is restricted to $A \sim A_0$ and the distributions of X are $P_\alpha(\ \mid D)$. Finally, if the class of distributions $P_\alpha(\ \mid D)$ is completed in a manner that leaves f well defined, the above may be iterated to construct admissible estimators. For the problems we consider, non-unique Bayes procedures occur exactly when the support of the a priori

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