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# CONVERGENCE RATES FOR EMPIRICAL BAYES TWO-ACTION PROBLEMS I. DISCRETE CASE

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**1. Introduction and summary.** Situations involving sequences of similar but independent statistical decision problems arise in many areas of application. Routine bioassay (Chase (1966)) and lot by lot acceptance sampling are typical examples of such situations. In many instances it is reasonable to formulate the independent component problems of such a sequence as Bayes statistical decision problems involving a common, but completely unknown, prior probability distribution over the state space. Robbins (1955) has shown for certain estimation problems that the accumulated information acquired as the sequence of problems progresses may be used to improve the decision rule at each stage. Such "empirical Bayes" procedures may be asymptotically optimal in the sense that the risk for the $n$th decision problem converges to the Bayes optimal risk which would have been obtained if the prior distribution were *known* and the best decision rule based on this knowledge were used.

Johns (1957) exhibits asymptotically optimal empirical Bayes procedures for certain two-action (hypothesis testing) problems as well as for estimation problems in a nonparametric context. Robbins (1963) and Samuel (1963) consider parametric two-action problems where the distributions of the observations are members of a specified exponential family, and where the special loss functions of Johns (1957) are used. Robbins and Samuel each exhibit asymptotically optimal empirical Bayes procedures for both discrete and continuous observations.

The usefulness of empirical Bayes procedures in practical statistical applications clearly depends on the rapidity with which the risks incurred for the successive decision problems approach the optimal limit. The purpose of this paper and its sequel (Johns and Van Ryzin (1967)) is to investigate rates of convergence to optimality of empirical Bayes procedures for two-action decision problems when the distributions of the observations are of exponential type. The present paper considers discrete exponential families which include for example the geometric, the negative binomial, and the Poisson distributions. The sequel (Johns and Van Ryzin (1967)) considers continuous exponential families with particular emphasis on the normal and the negative exponential distributions.

Each component problem in the sequence of decision problems for which an empirical Bayes procedure is to be defined is assumed to have the following

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