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REGULARITY OF EXCESSIVE FUNCTIONS II[1]

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1. Introduction. In [3] we introduced some conditions which are equivalent to the regularity of excessive functions under certain additional hypotheses. Unfortunately the additional hypotheses assumed in [3] are much too strong, and, in fact, can be eliminated completely. Thus the present paper represents a considerable extension and simplification of the results of [3]. Moreover, it may be read independently of [3].

All terminology and notation are the same as in [1] unless explicitly stated otherwise. In particular we fix once and for all a standard process $X = (\Omega, \mathscr{F}, \mathscr{F}_t, X_t, \theta_t, P^x)$ with state space $(E, \mathscr{E})$, and all stopping times are $\{\mathscr{F}_t\}$ stopping times unless explicitly stated otherwise. In Section 2 we characterize those stopping times which are accessible on $\{T < \zeta\}$—here accessibility is defined as in the general theory of processes, [4] or [7], and not as in [1]. See Section 2 for the precise definition. In the case of a special standard process our result reduces to a criterion of Meyer [6]. Even though all of the techniques needed for the construction in Section 2 are well known, we have given some details since the result seems to have been overlooked in the literature and is, perhaps. of some independent interest. In any case it is crucial for the discussion in Section 4. In Section 3 we introduce a topology on the state space E which for lack of a better name we call the d-topology. In Section 4 we relate this topology to the regularity of excessive functions. Roughly speaking, an excessive function is regular if and only if it is d-continuous. See Proposition 4.2 for the precise statement. Also we give a necessary and sufficient condition that all excessive functions be regular. See Proposition 4.4.

2. Accessibility of stopping times. Recall that a stopping time T is *accessible* if for each initial measure μ there exists a sequence $\{\Lambda_k\}$ of sets in $\mathscr{F}_T$ such that $\{T > 0\} = \cup \Lambda_k$ almost surely P^μ and for each k there exists an increasing sequence $\{T_n{}^k\}$ of stopping times bounded by T and such that almost surely P^μ, $\{T_n{}^k\}$ increases to T strictly from below on Λ_k; i.e., $\lim_n T_n{}^k = T$ and $T_n{}^k < T$ for all n almost surely P^μ on Λ_k. If T is a stopping time and $\Lambda \in \mathscr{F}_T$ we say that T is accessible on Λ provided T_Λ is accessible where $T_\Lambda = T$ on Λ and $T_\Lambda = \infty$ on Λ^c. It is easily seen that this is equivalent to the statement that for each μ, almost surely P^μ we have $\{T > 0\} \cap \Lambda = \cup \Lambda_k$ with each $\Lambda_k \in \mathscr{F}_T$ and with T the limit strictly from below on Λ_k of an increasing sequence of stopping times. In general we use the terms accessible, totally inaccessible, and previsible as in the general theory of processes

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