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A LOCALLY MOST POWERFUL TIED RANK TEST IN A WILCOXON SITUATION[1]

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1. Introduction. Various ways of treating ties have been suggested and investigated, but little is known about tied rank tests which are optimal against specific alternatives. To my knowledge the only results in this field were achieved by Vorlíčková (1970). The author makes use of the concept of contiguity and generalizes the corresponding theorems of Hájek, Šidák (1967) to the case of rv's which take the values $k = 0, \pm 1, \pm 2, \cdots$ only. She proves that tests based on linear rank statistics are asymptotically most powerful against suitably chosen alternatives, if the method of averaged scores is used. Considering a discrete analogue of the logistic distribution one may derive in this way the two-sample Wilcoxon midrank test as an asymptotically most powerful test.

This paper presents a general framework for constructing optimal tied rank tests in the two-sample problem.[3] In particular, we will consider a class of discrete alternatives, analogous to some continuous alternatives against which the Wilcoxon test is locally most powerful, and construct a corresponding locally most powerful tied rank test. The test statistic is the sum of ranks obtained by ranking the distinct values in the pooled sample.

2. A general procedure for obtaining optimal tied rank tests. We assume that all occuring discrete distributions are lattice distributions on a real lattice

$$M = M(\xi_0, \delta) = \{\xi_k : \xi_k = \xi_0 + \delta k, k = 0, \pm 1, \pm 2, \cdots\}, \qquad \delta > 0,$$

or on a set M' derived from M by applying any continuous and strictly increasing transformation of the real line onto itself respectively.

Considering $X_{11}, \cdots, X_{1n_1}$ independent rv's with df F_1, and $X_{21}, \cdots, X_{2n_2}$ with F_2, we want to test $F_1 = F_2$ against $F_1 \leqq F_2$. ($F_1 \leqq F_2$ means, that $F_1(x) \leqq F_2(x)$ holds for all real x and this strictly for at least one x.) We get in the usual way $R(x) = (R_1(x), R_2(x)) = (r_{11}, \cdots, r_{1n_1}, r_{21}, \cdots, r_{2n_2})$ as a maximal invariant statistic relative to the group of the above mentioned transformations. The ranks are defined as follows: In the pooled sample $x = (x_{11}, \cdots, x_{2n_2})$ tied observations are regarded as one observation. Let T denote the size of the reduced sample x'. For x'

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[3] In Krauth (1969) one-sided permutation and rank tests for the matched pairs problem, and the problem of independence are similarly derived.