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# THE RANDOMIZATION MODEL FOR INCOMPLETE BLOCK DESIGNS

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**1. Introduction.** The randomization scheme for incomplete block designs consists of assigning treatments to a plan of blocks and plots and allocating the block positions and the positions of plots within blocks randomly to the experimental units. It is the purpose of this paper to consider the asymptotic properties of estimates of treatment contrasts and of certain permutation tests under the randomization models appropriate to these designs. Similar results have been obtained for the randomized block design by Wald and Wolfowitz (1944) and for the completely randomized design by Silvey (1954).

In Section 2, two theorems are derived, giving the limiting distribution of linear forms in the universe of restricted permutations obtained by the randomization procedure for incomplete block designs. These theorems are applied in Section 3, to show that the permutation distributions of certain test statistics have the same limiting form as their limiting distributions under the usual normal theory models. In Section 4, combined estimation of treatment contrasts, using both intra-block and inter-block information, is considered and the limiting distribution of a combined test statistic is obtained.

**2. Two combinatorial limit theorems.** Let $c_{nij}, a_n(i, j)$ $(i = 1, \cdots, n, j = 1, \cdots, k)$ be $2kn$ real numbers defined for every positive integer $n$. Let $(I_{n1}, \cdots, I_{nn})$ be the random variable taking each permutation of $(1, \cdots, n)$ with equal probability and let $(J_{i1}, \cdots, J_{ik})$, $(i = 1, \cdots, n)$ be $n$ independent random variables each taking each permutation of $(1, \cdots, k)$ with equal probability independently of $(I_{n1}, \cdots, I_{nn})$. We consider the asymptotic distribution of certain random variables of the form

$$L_n = \sum_{i=1}^n \sum_{j=1}^k c_{nij} a_n(I_{ni}, J_{ij}) , \tag{1}$$

as $n$ tends to infinity.

Write $\mathbf{I}_n = (I_{n1}, \cdots, I_{nn})$. We can show immediately that

$$E(L_n \mid \mathbf{I}_n) = k \sum_{i=1}^n c_{ni\cdot} a_n(I_{ni}, \cdot) \tag{2}$$

and then that

$$E(L_n) = nkc_{n\cdot\cdot} a_n(\cdot, \cdot) \tag{3}$$

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