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PLAY-THE-WINNER RULE AND INVERSE SAMPLING FOR SELECTING THE BEST OF $k \geqq 3$ BINOMIAL POPULATIONS

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1. Introduction.

Given. k independent binomial populations with unknown probabilities p_i' of success and q_i' of failure ($p_i' + q_i' = 1$) on a single trial ($i = 1, 2, \cdots, k$).

Problem. to select the (or a) best population (i.e., the one with the largest p', say p_1).

Main emphasis. the comparison of procedures (all using inverse-sampling stopping rules) that differ only in the sampling method.

The first procedure, R_I, uses play-the-winner, cyclic variation (PWC) sampling rule. It puts the populations in a random order at the outset say $\pi_1, \pi_2, \cdots, \pi_k$. Population π_j is sampled until a failure is observed and then π_{j+1} is sampled ($j = 1, 2, \cdots, k$); π_{k+1} is identified with π_1. Sampling terminates as soon as any one population has r successes; that population is selected as best. We determine r so that the probability of a correct selection (CS) satisfies

$$P\{\text{CS} \mid R_I\} \geqq P^* \quad \text{whenever} \quad p_1 - \max_{j>1} p_j \geqq \Delta^* , \tag{1.1}$$

where the constants P^* (with $1/k < P^* < 1$) and Δ^* (with $\Delta^* > 0$) are preassigned. Approximations and a table for $r = r(P^*, \Delta^*)$ are given for selected values of k, P^* and Δ^*. Table 2 gives exact vs. approximate expected total number of observations $E\{N \mid R_I\}$ for $k = 2$ and some comparisons with a fixed sample size procedure.

The second procedure, R_I', uses vector-at-a-time (VT) sampling; it takes one observation from each of the k populations (simultaneously) until at least one of them has r successes. The winner (or one selected from the winners at random) is then chosen as best. We determine r by (1.1) with R_I replaced by R_I'. It is shown (Section 4) that the minimum $P\{\text{CS}\}$ and hence the value of r required to satisfy (1.1) is exactly the same for the PWC-rule (procedure R_I) and the VT-rule (procedure R_I'). (In [5] a similar result was found for fixed-sample stopping rules; also see the discussion of procedure $\hat{R}_I$ below.)

A procedure, R_I^*, dual to R_I is studied (Section 6); it is based on waiting for a fixed number of failures and it is shown asymptotically to be an improvement on R_I when $p_1 < \frac{1}{2}$.

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