The Annals of Probability
2023, Vol. 51, No. 6, 2387–2395
https://doi.org/10.1214/23-AOP1650
Main article: https://doi.org/10.1214/20-AOP1482

ERRATUM TO "AN OPTIMAL REGULARITY RESULT FOR KOLMOGOROV EQUATIONS AND WEAK UNIQUENESS FOR SOME CRITICAL SPDES"

BY ENRICO PRIOLA[a]

Dipartimento di Matematica, Università di Pavia, [a]*enrico.priola@unipv.it*

This note corrects the main result in *Ann. Probab.* **49** (2021) 1310–1346

This note is a correction of [7] (more details can be found in https://arxiv.org/abs/2210.07096). The paper [7] deals with SPDEs like

$$dX_t = AX_t\,dt + (-A)^{1/2}F\big(X(t)\big)\,dt + dW_t, \quad X_0 = x \in H, \tag{1}$$

where $A : \mathrm{D}(A) \subset H \to H$ is a negative definite self-adjoint operator on a real separable Hilbert space H with inner product $\langle \cdot, \cdot \rangle$ having A^{-1} of trace class (see [7], Hypothesis 1); moreover, W is a cylindrical Wiener process on H. Theorem 1 in [7] asserts that if $F : H \to H$ is continuous with at most linear growth then, for any $x \in H$, there exists a weak mild solution defined on some filtered probability space. Moreover, uniqueness in law (or weak uniqueness) holds for any $x \in H$. The existence part of the theorem is proved in [7], Section 4. However, the proof of the uniqueness part is not correct.

Here, following [7], we prove weak uniqueness for (1), replacing the continuity assumption of F with the stronger assumption that F is *locally* θ-Hölder continuous for some $\theta \in (0, 1)$; see Theorem 1. The fact that F is locally θ-Hölder continuous allows to study singular perturbations of classical stochastic Burgers' equations (cf. Theorem 7 which extends Theorem 1). It remains an open problem if the weak uniqueness stated in [7], Theorem 1, holds.

The proof of the uniqueness part of [7], Theorem 1, used the following incorrect regularity result for the Kolmogorov equation $\lambda u - Lu = f$ associated to the SPDE when $F = 0$ ($\lambda > 0$, $f : H \to \mathbb{R}$ Borel and bounded): the first derivative $Du(x)$ belongs to $\mathrm{dom}((-A)^{1/2}) = D((-A)^{1/2})$, for any $x \in H$, and

$$\sup_{x\in H} \big|(-A)^{1/2} Du(x)\big|_H = \big\|(-A)^{1/2} Du\big\|_0 \le C \|f\|_0. \tag{2}$$

(C is independent of f; cf. [7], Theorem 7.) Note that [7], Lemma 6, was used to prove (2), but there is a mistake in the proof of such lemma (see Remark 2 for more details). On the other hand, [7], Theorem 7, does not hold in general (a counterexample is given in [4]). In this note to prove the uniqueness part of Theorem 1, we will replace (2) with the following two estimates involving Hölder norms:

$$\big\|(-A)^{1/2} Du\big\|_0 \le \frac{\Gamma(\theta/2)}{\lambda^{\theta/2}} \|f\|_{C^\theta}, \qquad \big[(-A)^{1/2} Du\big]_{C^\theta} \le M_\theta \|f\|_{C^\theta}; \tag{3}$$

see page 3 and Theorem 3 for more general estimates. Clearly, the first estimate in (3) is weaker than (2). Theorem 3 is an optimal regularity result in Hölder spaces similar to the one in [3].

Received October 2022; revised July 2023.
MSC2020 subject classifications. Primary 60H15, 35R60; secondary 35R15.
Key words and phrases. Critical SPDEs, weak uniqueness in infinite dimensions, optimal regularity for Kolmogorov operators.