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Rejoinder: Let's Be Imprecise in Order to Be Precise (About What We Don't Know)

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Preparing a rejoinder is a typically rewarding, sometimes depressing and occasionally frustrating experience. The rewarding part is self-evident, and the depression sets in when a discussant has much deeper and crisper insights about the authors' thesis than the authors themselves. Frustrations arise when the authors thought they made some points crystal clear, but the reflections from the discussants show a very different picture. We are deeply grateful to the editors of *Statistical Science* and the discussants for providing us an opportunity to maximize the first, sample the second and minimize the third.

1. LET'S AUGMENT OUR SHOES TO FIT OUR GROWING FEET

Professor Glenn Shafer's historically infused and theoretically fermented insights provided us with an intense savoring and much lingering. His succinct summary of the three branches of the art of conjecture of d'Alembert laid out the contours and interplay among (precise) probability, statistics and imprecise probability. The first branch enters the game of conjecture by manipulating theoretically precisely specified quantities and models, a game of precise probability, deducing properties and consequences of a theoretical construct.

The second branch plays the same game empirically, by focusing on assessing chances and risks from data. This captures the essences of the current statistical practices, when empirical assessments are guided by the rules of precise probability. Principled statistical practices fully recognize the multiple uncertainties in empirical assessments, and hence have built-in risk assessments for estimating the part of uncertainties that can be reasonably gauged empirically. For parts that cannot be empirically assessed internally, sensitivity studies have been the primary tool, precisely because by posting specific alternative scenarios, we can traverse within the first two branches, and hence remain in our comfort zone.

Shafer's summary made clear that the third branch clamors more attention than we currently bestow. This branch covers the vast majorities of inquires where precise probabilistic descriptions, whether theoretical or empirical, are inherently incomplete or impossible. In our own applied work, when we ask a subject expert to provide a prior, the most precise answer would be of the kind "I'm quite sure that α is between 1 and 2." Any further inquiry about how α is distributed on $[1, 2]$ would be met with either a puzzled face or an answer few of us like: "I have no idea."

Such "no-idea" answers have motivated many to work harder throughout history. Nevertheless, currently we are still forced to make up assumptions, such as α is distributed uniformly on $[1, 2]$, for the sole purpose of applying available theories or methods. Or as Shafer put it, despite efforts to move bits of the third branch into the first two, "the third still seems very large."

Instead of cutting feet to fit shoes, the framework of *imprecise probability* (IP) suggests a less painful paradigm: expanding the shoes to fit the feet. This metaphor has another leg to stand on because the imprecise shoes are no less functional than the precise ones. As Augustin and Schollmeyer emphasized, IP should have been better named as "set-valued probability." But sound statistical inference is already set-valued, as classical paradigms have delivered via confidence intervals and Bayesian credible sets, in order to reflect inferential uncertainty. In that sense, the set-valued output of IP models is no less familiar a mathematical form than that from precise probability models, albeit carrying a different interpretation of "uncertainty". It is therefore natural for us to ask: why can't we go from set-valued input to set-valued output directly, instead of squeezing through the narrow tunnel of numerically valued probability?

2. TWO CONCERNS THAT MOTIVATED OUR WORK

To answer this question, we would like to elaborate our view on the role of imprecise probabilities and their accompanying updating rules. We surmise nearly all statisticians take for granted that probability is the language of uncertainty. And by probability, we specifically mean countably additive probability that obeys the Kolmogorov axioms. Bayesians, Frequentists, as well as those who entertain fiducial, structural and functional inference, all operate within a framework that guides the expression of uncertainty relating observable information to unknown

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