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Errata for *Perturbation by non-local operators*

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There is a gap in the proof of (3.19) in [1, Theorem 3.6] in that the constant C_{14} in [1, (3.22)] depends on $r^{1/\alpha}\lambda$ rather than $\lambda > 0$ and so when applying [1, Lemma 3.4] it gives a new A_0 depending also on r. This gap affects only the proof of (1.16) of [1, Theorem 1.1(v)] (or [1, (3.23)]). The rest of [1, Theorem 3.6] including the estimates (3.20)–(3.21), (3.6) and (3.8) hold without any issue. The proof of (3.19) in [1, Theorem 3.6] works if we drop λ and replace $M_{b,\lambda}$ defined in [1, (1.13)] by $\|b\|_\infty$.

In this errata, instead of establishing [1, (3.19)], we show directly that the estimate (1.16) of [1, Theorem 1.1(v)] hold for every $\lambda > 0$. We point out that all the main results stated in the Introduction of [1] remain true.

First note that by Lemma 0.1 below, Lemmas 3.1 and 3.4, Theorems 3.6 and 3.7 of [1] hold for $\lambda = +\infty$ with (3.2), (3.11), (3.12), (3.19) and (3.23) being replaced by

$$\left|q^b\right|_n(t,x,y) \le C_{11}\left(\|b\|_\infty C_7 c\right)^n g_1(t,x,y), \quad t \in (0,T], x, y \in \mathbb{R}^d, \tag{3.2'}$$

$$\left|q^b_{n+1}(t,x,y)\right| \le C_{13} 2^{-n} \|b\|_\infty p_1(t,x,y) \quad \text{for } t \in (0,1] \text{ and } x, y \in \mathbb{R}^d, \tag{3.11'}$$

$$\left|\mathcal{S}^b_x q^b_n(t,x,y)\right| \le C_{12} \|b\|_\infty 2^{-n} f_0(t,x,y) \quad \text{for } t \in (0,1] \text{ and } x, y \in \mathbb{R}^d, \tag{3.12'}$$

$$\left|q^b_n(t,x,y)\right| \le C_{14} 2^{-n}\left(t^{-d/\alpha} \wedge \left(\frac{t}{|x-y|^{d+\alpha}} + \frac{\|b\|_\infty t}{|x-y|^{d+\beta}}\right)\right) \tag{3.19'}$$

and

$$\left|q^b(t,x,y)\right| \le 2C_{14}\left(t^{-d/\alpha} \wedge \left(\frac{t}{|x-y|^{d+\alpha}} + \frac{\|b\|_\infty t}{|x-y|^{d+\beta}}\right)\right), \tag{3.23'}$$

respectively, where the constant c is the one in Lemma 0.1 and that the constant A_0 in [1, Lemma 3.4] can be chosen to be smaller than $1/(2C_{12})$. This gives the existence and uniqueness of the fundamental solution $q^b(t,x,y)$ and all the stated properties in [1, Theorem 1.1] except that we need to replace $p_{M_{b,\lambda}}$ by $p_{\|b\|_\infty}$ in the estimate [1, (1.16)].

For $a \ge 0$, denote by $p_a(t,x,y)$ the fundamental solution of $\Delta^{\alpha/2} + a\Delta^{\beta/2}$. Recall that for each $\lambda > 0$ and $a \ge 0$, $f_{a,\lambda}(t,x,y)$ is defined as in [1, (2.6)], and that $f_{a,\infty}(t,x,y) = f_0(t,x,y)$, which is given by [1, (2.1)].

By a similar argument as [1, Lemma 2.5], one obtains the following inequality.

Lemma 0.1. *There exists $c = c(d,\alpha,\beta) > 0$ such that for all $t \in (0,1]$ and $x, y \in \mathbb{R}^d$,*

$$\int_0^t \int_{\mathbb{R}^d} p_1(t-s,x,z) f_0(s,z,y)\, dz\, ds \le c p_1(t,x,y).$$