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Editorial: Special Issue on "Nonparametric Inference Under Shape Constraints"

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1. INTRODUCTION

Shape-constrained inference usually refers to nonparametric function estimation and uncertainty quantification under qualitative shape restrictions such as monotonicity, convexity, log-concavity and so on. One of the earliest contributions to the field was by Grenander (1956). Motivated by the theory of mortality measurement, he studied the nonparametric maximum likelihood estimator of a decreasing density function on the nonnegative half-line. A great attraction of this estimator is that, unlike other nonparametric density estimators such as histograms or kernel density estimators, there are no tuning parameters (e.g., bandwidths) to choose.

Over subsequent years, this idea has been extended and developed in many different directions. On the applied side, there has been a gradual realisation that nonparametric shape constraints are very natural to impose in many situations. For instance, monotonicity of a regression function arises in many contexts such as genetics (Luss, Rosset and Shahar, 2012), medicine (Schell and Singh, 1997) and dose-response modelling (Lin et al., 2012). Shape-constrained procedures are also commonly used in economics (Matzkin, 1991, Varian, 1984) and survival analysis, for instance in the interval-censoring problem and hazard function estimation; see the recent book by Groeneboom and Jongbloed (2014). Many other applications, and further developments, including the computational aspects of these shape-constrained estimators, are nicely summarised in the books by Barlow et al. (1972), Robertson, Wright and Dykstra (1988) and Groeneboom and Wellner (1992).

On the theoretical side, it has been known since the work of Prakasa Rao (1969) that the Grenander estimator exhibits nonstandard asymptotic behaviour (more precisely, it converges at rate $n^{-1/3}$, where n is the sample size, at points at which the true decreasing density is differentiable with negative derivative). Moreover, Groeneboom (1985) obtained the limiting distribution of the L_1-distance between the Grenander estimator and true density. The study of the likelihood ratio test for monotone functions was initiated by Banerjee and Wellner (2001), while the adaptive behaviour of monotonicity-constrained estimators was highlighted by Birgé (1989) and Zhang (2002), using finite-sample risk bounds.

However, since the turn of the millennium (and the last decade in particular) the area of shape constraints has witnessed substantially increased activity. On the one hand, researchers started studying systematically the behaviour of univariate shape-constrained procedures beyond monotonicity, for instance in convexity-constrained models (Groeneboom, Jongbloed and Wellner, 2001) and log-concave density estimation (Dümbgen and Rufibach, 2009, Balabdaoui, Rufibach and Wellner, 2009). On the other hand, there has been a realisation that shape-constrained methods have much to offer in multi-dimensional problems (e.g., Cule, Samworth and Stewart, 2010, Seijo and Sen, 2011, Koenker and Mizera, 2010, Han et al., 2018, Seregin and Wellner, 2010). The scope of the field has been broadened by the emergence of new applications, including convex set estimation (Brunel, 2013, Guntuboyina, 2012, Gardner, Kiderlen and Milanfar, 2006, Gardner, 2006), shape-constrained dimension reduction (Chen and Samworth, 2016, Xu, Chen and Lafferty, 2016, Groeneboom and Hendrickx, 2018) and ranking and pairwise comparisons (Shah et al., 2017). New theoretical tools have been developed that have allowed us to make progress in understanding how shape-constrained procedures behave (Dümbgen, Samworth and Schuhmacher, 2011, Kim and Samworth, 2016, Cai and Low, 2015, Guntuboyina and Sen, 2013). Last but not least, increased computing power together with algorithmic advances mean that certain estimators have become computationally feasible (Koenker and Mizera, 2014, Mazumder et al.,

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