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Rejoinder: A Review of Self-Exciting Spatio-Temporal Point Processes and Their Applications

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I would first like to thank the discussants for their interesting and insightful comments. I was first motivated to write a review when I noticed the diversity of applications of self-exciting processes and the interesting innovations being made in widely separate fields, so it has been rewarding to read such a broad range of perspectives from the discussants.

I broadly agree with many of the points brought up by the discussants, so here I would like to focus on a few topics that point to areas of future research.

1. CAUSALITY AND MUTUALLY EXCITING PROCESSES

Professor Ogata raises an important topic otherwise omitted from my review: "the modeling of point processes for causality analysis from a stochastic process including another point process." He discusses models whose intensity function depends not just on the past history of the process but the past history of *another* process, for example, models of earthquake events incorporating stress change or fault-weakening events, and the use of AIC and other techniques to perform model selection and test hypotheses about earthquake precursors.

This resembles the "leading indicators" incorporated by Mohler (2014) to allow events such as disorderly conduct and public drunkenness to be used as predictors for more serious crimes, though Mohler did not perform extensive inference on these leading indicators to test specific hypotheses. I agree with Ogata that modeling of such leading indicators is essential for answering important questions in many areas of application, and would like to note some recent theoretical developments which may help with the task.

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If we have multiple separate processes, such as earthquake events and precursor events, or records of several different types of crimes, we can consider them as a single multivariate point process. As noted in Section 2.3 of the review, a multivariate point process is simply a marked point process in which the mark space is a finite set $\{1, \dots, m\}$. In a mutually exciting multivariate process, instead of a single triggering function g, there is a matrix of functions g_{ij} specifying the effect of event type j on the intensity of process i.

There has been a recent surge in methods for detecting causality in multivariate point processes, though largely focused on purely temporal processes. Eichler, Dahlhaus and Dueck (2017) recently demonstrated that the triggering functions g_{ij} have a direct connection to Granger causality: events of type i do not Granger-cause events of type j if and only if $g_{ji}(t) = 0$ for all $t \in \mathbb{R}$. This can be used to define a Granger causality graph G whose directed edges satisfy the property

$$i \to j \notin G \quad \Longleftrightarrow \quad g_{ji}(t) = 0 \quad \text{for all } t \in \mathbb{R},$$

indicating the Granger causality relationships between the mutually exciting processes.

There are now several competing methods for estimating the causality graph G and testing hypotheses about its edges [e.g., Chen, Witten and Shojaie (2017), Xu, Farajtabar and Zha (2016), Achab et al. (2017)], again focusing on purely temporal processes. It would be quite interesting to see these methods extended to spatio-temporal mutually exciting processes and used in applications, where G may answer substantive scientific questions and the use of leading indicator processes could improve predictions.

2. STOCHASTIC RECONSTRUCTION

Professor Zhuang points out that the goodness-of-fit methods given in the review evaluate the fit of the entire model, either on all the data or over specific space-time regions (like the residual methods of Section 3.6), leaving an important gap: it may be necessary to assess