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Comment on "A Review of Self-Exciting Spatio-Temporal Point Process and Their Applications" by Alex Reinhart

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I believe that Dr. Reinhart has written an excellent review on the methodologies, techniques and applications related to spatio-temporal self-exciting processes that have developed during recent years. Here, I would like to mention the following points to complement this article:

1. *On the diagnostics related to the clustering model*. All the methods that are explained in this article give the goodness-of-fit of the entire model, globally to the data or in a local window of the observation space–time range. To check whether the formulation for each individual component is appropriate for fitting the data or not, the stochastic reconstruction techniques (Zhuang, Ogata and Vere-Jones, 2004), based on which the diagnostics of each model components can be easily constructed, can be utilized. Zhuang, Ogata and Vere-Jones (2004) and Zhuang (2006) also used this method to test some hypotheses that are related to earthquake clusters but not formulated in the model. The method is helpful for finding the clues of formulating better models.

2. *On estimating background rate.* Not only separable clustering structure discussed in this review paper, but also complex and nesting background components can be reconstructed. Recently, Zhuang and Mateu (2018) developed a semiparametric estimation method to obtain simultaneously the clustering structure and the background in the occurrence rate of crimes, where the latter includes two periodic components, a daily and a weekly. The estimation procedure can be outlined as following.

Consider a model with a conditional intensity function

$$\lambda(t,x) = \mu_t(t)\mu_d(t)\mu_w(t)\mu_b(x) + \iint_{(-\infty,t)\times S} g(t-s, x-u)\,dN(s,u),$$

where $\mu_t(t)$, $\mu_d(t)$ and $\mu_w(t)$ represent the trend term, the daily periodicity and the weekly periodicity in the temporal components of the background rate, respectively, $\mu_b(x)$ represents the spatial homogeneity of the background rate, and $g(t-s, x-u)$ represents the subprocess triggered by an event previously occurring at location u and time s.

Given a realization of the point process $\{(t_i, x_i) : i = 1, 2, \dots, n\}$ in a time-space range $[T_1, T_2] \times S$, where t_i and x_i denote the occurrence time and location, respectively, the long-term trend term $\mu_t(t)$ in the background component can be reconstructed in the following way. Let

$$w^{(t)}(t,x) = \mu_t(t)\mu_b(x)/\lambda(t,x).$$

Then, assuming that μ_t is smooth enough,

$$\begin{aligned}\sum_i & w^{(t)}(t_i,x_i)\mathbf{1}(t_i \in [t-\Delta_t, t+\Delta_t]) \\ &\approx \int_{T_1}^{T_2} \iint_S w^{(t)}(s,x)\lambda(s,x) \\ &\quad \cdot \mathbf{1}(s \in [t-\Delta_t, t+\Delta_t])\,ds\,dx \\ &= \int_{t-\Delta_t}^{t+\Delta_t} \mu_t(s)\,ds \iint_S \mu_b(x)\,dx \\ &\propto \int_{t-\Delta_t}^{t+\Delta_t} \mu_t(s)\,ds \\ &\approx 2\mu_t(t)\Delta_t,\end{aligned}$$

where Δ_t is a small positive number. That is,

$$\hat{\mu}_t(t) \propto \sum_i w_i^{(t)} \mathbf{1}(t_i \in [t-\Delta_t, t+\Delta_t]),$$

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