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Correction to "A Topologically Valid Definition of Depth for Functional Data"

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We are grateful to Irène Gijbels and Stanislav Nagy for drawing our attention to some regrettable substantive errors in our paper, which appears in *Statistical Science* **31** 61–79 (2016). With apologies, we present the correct forms below.

1. In Definition 3.1 and in the definitions of band depth and modified band depth on page 68 (lines 15–16, 27–28 and 34) α should be replaced by $\alpha(v)$.
2. The last two lines of Definition 3.1 should be replaced by $U : \mathcal{V} \to \mathfrak{F}$ with $U(v) := \sup_{x\in\mathcal{E}} x(v)$ and $L : \mathcal{V} \to \mathfrak{F}$ with $L(v) := \inf_{x\in\mathcal{E}} x(v)$ when $\max(|U(v)|, |L(v)|) < \infty$ for all $v \in \mathcal{V}$.
3. In Definition 3.2:
   - under P-3., after "exists" should appear "with $D(z, P) = D(z', P)$ implying $d(z, z') = 0$".
   - Equation (3.1) has to be substituted by $\sup_{y\in\mathfrak{F}_x:d(x,y)<\delta} D(y, P) \le D(x, P) + \varepsilon$, where $\mathfrak{F}_x := \{y \in \mathfrak{F} : d(y,x) < d(y,\theta)$ or $\max\{d(y,\theta), d(y,x)\} < d(x,\theta)\}$ for $\theta = \operatorname{argsup}_{x\in\mathfrak{F}} D(x, P)$.
   - In P-5., $\mathfrak{C}(\mathfrak{F}, P)$ is substituted by $\mathfrak{C}(\mathfrak{F}, P) \setminus 0$ and the interval of definition of δ by $[\inf_{v\in\mathcal{V}} d(L(v), U(v)), d(L, U)) \cap (0, \infty)$.
4. Lemma 4.3 is false. Consequently, there is a cross in the corresponding position in Table 2.

   Counter-example: Let $(\mathfrak{F}, d) = (\mathbb{H}, \|\cdot\|_{\mathbb{L}_2})$ and P a discrete distribution on $\mathfrak{F}$ with support $\{X_1, X_2, X_3\}$ such that $P(X_1) = P(X_3) = 1/4$ and $P(X_2) = 1/2$ and $d(X_1, X_2) = d(X_2, X_3) = d(X_1, X_3)/2$. Let $x \in \mathfrak{F}$ such that $d(x, X - 1) = d(x, X_2) = d(X_1, X_2)/2$, then, there always exist a h such that $D_h(x, P) < \min(D_h(X_1, P), D_h(X_2, P))$. This is due to $D_h(x, P) = \mathbb{E}[\exp(-(x/h)^2/2)/(h\sqrt{2\pi})]$.

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