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Rejoinder: Approximate Models and Robust Decisions

James Watson and Chris Holmes

We wish to thank all of the discussants for their insightful comments. We have certainly benefitted from considering their perspective of our work. In the following rejoinder, we begin by reiterating the central tenet of our approach, followed by some general pointers to common themes arising across the discussions, and finally a point-by-point reply to some specific issues raised by individual reviewers.

The overriding objective of our work is to advocate the inclusion of decision analysis within the iterative process of scientific learning as laid out in the seminal paper of Box (1980). This iterative process proceeds firstly by a model *estimation* stage where the statistical model is updated *as if* it was true. This update, for us, takes the form of a Bayes posterior. In the approach of Box, the modeller then undertakes a second stage of model *criticism* that potentially leads to model adjustments, for example, via model elaboration, followed by re-estimation, re-criticism and so forth. In our paper, we call for the use of formal and informal (exploratory) decision analysis into the model criticism stage of Box that directly takes into account the context and rationale for the model's use and the questions under consideration.

It is important to note that we are not advocating $\pi^{\sup}_{a,C}$ as a true model for the data, nor necessarily an actual representation of beliefs, although it is interesting to see connections with the historic use of robust priors (Section 4.2). As stated by Box, model criticism and parameter estimation are distinct and demand different methodologies. It is $\psi^{\sup}_{a,C}$, the maximum expected loss occurring within a KL neighbourhood of size C at the model criticism stage that is our fundamental object of interest. For if $\psi^{\sup}_{a,C}$ is relatively large for small changes to the estimated model, then it indicates that the downstream decision analysis may be highly unstable, as for small changes in the posterior model we can observe a substantial increase in expected loss. Highlighting this at the model criticism stage allows for further diagnostics, insight and model elaboration. We believe it is incumbent on modellers to take into account the context of their models and where appropriate incorporate decision analysis into their model criticism.

Now beginning from this standpoint we consider some general themes that arose from the discussants before considering whether or not models should be discarded all together, the setting of the KL neighbourhood size and ending our rejoinder with answers to some finer points.

1. EX POST CRITICISM

1.1 Formal Model Checking: Moving Away from "Statistical Truths"

The discussion by economists Hansen and Marinacci (H&M) provides an interesting and refreshing perspective on model misspecification from outside of traditional statistics. They espouse Wald's philosophy of decision making, where the goal is no longer discovering "statistical truths" but rather to use models operationally in light of a posited objective function (loss function).[1] H&M show how concerns of model misspecification can be expressed as "aversion to ambiguity" as stated in its general form as an optimization problem where the decision maker solves

$$\max_{a\in A}\min_{\pi}\int_{\Theta} U_a(\theta)\pi(\mathrm{d}\theta) + C(\pi), \tag{1}$$

where $U_a(\theta) = -L_a(\theta)$ is the utility function, C is a penalty function, or regularization term, that encodes for *variational preferences* over probability models (Maccheroni, Marinacci and Rustichini, 2006). The form of (1) is instructive, as for KL divergence (relative entropy) and other convex C the solution of (1)

James Watson is a Postdoctoral Researcher at the Mahidol-Oxford Research Unit, Centre for Tropical Medicine and Global Health, Nuffield Department of Clinical Medicine, University of Oxford, Oxford, United Kingdom (e-mail: jwatowatson@gmail.com). Chris Holmes is Professor of Biostatistics, Department of Statistics, University of Oxford, United Kingdom (e-mail: cholmes@stats.ox.ac.uk).

[1] Pascal's wager is the most famous example of this.