

## INTRODUCTION TO MEMORIAL ISSUE FOR DONALD BURKHOLDER (1927–2013)

Don Burkholder had a long and distinguished career in mathematics. He is recognized worldwide for his deep and lasting contributions to martingale theory and its applications to other areas of mathematics. As a person, colleague and friend, he was kind and generous, serving as mentor and role model for many young people entering the field. Burkholder published many of his foundational papers in IMS journals, particularly *The Annals of Mathematical Statistics* and *The Annals of Probability*. His many contributions to probability and analysis have been reviewed in three recent publications: (1) “Don Burkholder’s work on Banach spaces,” (2) “Donald Burkholder’s work in martingales and analysis” and (3) “The foundational inequalities of D. L. Burkholder and some of their ramifications.” The first two articles, one written by Gilles Pisier and the other by Rodrigo Bañuelos and Burgess Davis, respectively, appeared in “*Selected Works of Donald L. Burkholder*,” published by Springer in 2010. The third article, written by Bañuelos, appeared in the special volume *Don Burkholder: A collection of articles in his honor*, of the *Illinois Journal of Mathematics*, 2011. The reader is referred to these articles for an in-depth look at Burkholder’s work, its applications and connections to different fields in mathematics where his work has had, and continues to have, an impact. Here, we briefly touch upon a few of his papers.

In 1966, Burkholder published his celebrated “Martingale Transforms” paper in *The Annals of Mathematical Statistics*. In the 1930s, Marcinkiewicz and Paley (in separate papers) proved an inequality for the Haar system of functions in the unit interval, which is equivalent to the boundedness of dyadic martingale transforms with the predictable sequence taking values in  $\{1, -1\}$ . Burkholder’s paper extended this result to the general setting of martingales. This paper was inspired by the seminal work of Donald Austin, published in the same volume of *The Annals of Statistics*, which showed the finiteness of the square function of  $L^1$  bounded martingales. The groundbreaking paper, “Extrapolation and Interpolation of Quasi-Linear Operators on Martingales,” written with Richard Gundy, was published in *Acta Mathematica* in 1970. The paper introduced the “good  $\lambda$  method,” now important in many areas of mathematics for comparing norms of operators, and used it to prove the famous Burkholder–Gundy martingale inequalities which compare the norms of the square and maximal functions for all continuous path martingales and many other regular martingales. In a subsequent paper Burkholder, together with Gundy and Martin Silverstein, used these inequalities to solve a longstanding open problem of Hardy and Littlewood concerning conjugate functions. This revolutionary paper inspired many harmonic analysts to learn probability and many probabilists to learn harmonic analysis, greatly enriching both fields.