The Annals of Probability
2017, Vol. 45, No. 3, 2056–2058
DOI: 10.1214/16-AOP1096

ERRATUM TO “SCALING FOR A ONE-DIMENSIONAL DIRECTED POLYMER WITH BOUNDARY CONDITIONS”

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Annals of Probability (2012) **40** 19–73

The published version contains two errors which are corrected in a new version [1] posted on the arXiv and also on the author’s homepage.

The published version has a mistake on lines 3–5 of page 52. Namely, the reversal mapping has to be applied in a fixed rectangle, but here the rectangle varies with k.

This issue can be circumvented with a coupling of polymer paths that gives the following inequality (this is Lemma 5.4 in the corrected version).

LEMMA 1. *For each fixed* ω, $Q^{\omega}_{m_1,n}(\xi_x > 0) \le Q^{\omega}_{m_2,n}(\xi_x > 0)$ *for all* $0 < m_1 < m_2$ *and* $n \ge 0$.

PROOF. Fix ω. We construct a coupling of polymer paths. On the full lattice $\mathbb{Z}^2_+$, define a backward Markov kernel:

$$\overleftarrow{\pi}_{x,x-e} = \frac{Y_x Z_{x-e}}{Z_x} = \frac{Z_{x-e}}{Z_{x-e_1} + Z_{x-e_2}}, \qquad x \in \mathbb{N}^2, e \in \{e_1, e_2\},$$

with the obvious degenerate transitions $\overleftarrow{\pi}_{(i,0),(i-1,0)} = \overleftarrow{\pi}_{(0,j),(0,j-1)} = 1$ on the axes and absorption $\overleftarrow{\pi}_{(0,0),(0,0)} = 1$ at the origin. For each $x \in \mathbb{Z}^2_+ \setminus \{(0,0)\}$, pick a jump to $v(x) \in \{x - e_1, x - e_2\}$ according to these transition probabilities. Fix an endpoint (m,n). Construct a path $x_{0,m+n}$ from the origin to (m,n) backwards, beginning with $x_{m+n} = (m,n)$ and then iterating $x_k = v(x_{k+1})$ for $k = m+n-1, m+n-2, \dots, 0$. The process ends at $x_0 = 0$. The probability of the path is

$$\prod_{k=1}^{m+n} \overleftarrow{\pi}_{x_k,x_{k-1}} = \frac{1}{Z_{m,n}} \prod_{k=1}^{m+n} Y_{x_k} = Q^{\omega}_{m,n}(x_{0,m+n}).$$

In other words, specifying the jumps $\{v(x)\}$ constructs a simultaneous realization of the polymer paths under all quenched measures $Q^{\omega}_{m,n}$ for a fixed ω.

Suppose $m_1 < m_2$ and the path between the origin and (m_1, n) goes through the point $(1, 0)$. Then the same is true for the path between the origin and (m_2, n).

Received September 2015.