

**ADDENDUM TO
“OPTIMAL STOPPING UNDER MODEL UNCERTAINTY:
RANDOMIZED STOPPING TIMES APPROACH”**

BY DENIS BELOMESTNY^{*,†} AND VOLKER KRÄTSCHMER^{*}

Duisburg-Essen University^{} and National Research University Higher
School of Economics[†]*

1. Introduction. In the authors’ paper [2], the optimal stopping problems of the form

$$(1.1) \quad \sup_{\tau \in \mathcal{T}} \rho_0^\Phi(-Y_\tau)$$

were studied. Here, $(Y_t)_{t \in [0, T]}$ denotes a nonnegative stochastic process on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{0 \leq t \leq T}, \mathbb{P})$ with $T \in]0, \infty[$, \mathcal{T} stands for the set of (\mathcal{F}_t) stopping times $\tau \leq T$ and the functional ρ_0^Φ is a divergence risk measure w.r.t. a lower semicontinuous convex mapping $\Phi : [0, \infty[\rightarrow [0, \infty]$ (see [2] for a precise definition and further details). Meanwhile, we have realized that some assumptions in [2] are unnecessarily strong. More precisely, we required \mathcal{F}_t to be countably generated for any $t > 0$. However, this excludes many widely used filtered probability spaces like standard augmentations of the filtered probability spaces generated by general multidimensional diffusions. A key point in [2] is the so-called derandomization result (see Proposition 6.3 in [2]), which shows that we obtain the same optimal value for the stopping problem (1.1) if we enlarge the set of stopping times to randomized stopping times. A crucial step in the proof of Proposition 6.3 is Lemma 7.4 which in turn uses an argument from the theory of angelic spaces (see Proposition B.1). This argument relies on the assumption that (\mathcal{F}_t) is countably generated for $t > 0$. In the meantime, we have realized that this line of argumentation continues to hold true if we only require that for any $t > 0$, the L^1 -space $L^1(\Omega, \mathcal{F}_t, \mathbb{P}_{|\mathcal{F}_t})$ is weakly separable, that is, the weak topology on $L^1(\Omega, \mathcal{F}_t, \mathbb{P}_{|\mathcal{F}_t})$ is separable. The latter assumption turns out to be much weaker than the original one.

The aim of this addendum is to reformulate the main results of [2] under weakened assumptions on the filtration (\mathcal{F}_t) . In particular, we show that the case of augmented filtration generated by a right-continuous stochastic process is now encompassed.

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