

## Comment on Article by Dawid and Musio<sup>\*,†</sup>

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Dawid and Musio present interesting results on how to affect model comparison using proper scoring rules, focusing chiefly on Bayesian model comparison. Among the reasons stated to justify the proposed approach we note:

1. The insensitivity of the procedure to a renormalization of the prior distribution,
2. The flexibility and/or robustness of the method when implemented using a prequential score.

The focus of the article is on the derivation of consistency results for the proper scoring rule methods based both on their implementation through a multivariate score and a prequential score. There are very many such results in the article, but the gist of the argument is that some form of proper scoring rule method can produce a consistent procedure even in cases when the standard Bayesian approach fails to do so or when it fails altogether, as is the case when improper priors are used and Bayes factors cannot be calculated.

Consistent model selection is unquestionably a desirable property as is the formulation of a coherent, universal framework for statistical inference. The Bayesian approach using *proper* priors accomplishes the latter. The proposed proper scoring rule methods mend the complications that arise when the Bayesian approach is used with improper priors. However, the beauty of the coherent Bayesian inferential framework is lost when model comparison is no longer based on the likelihood score. As in all compromises, something is gained at the expense of losing something else, or, as some would say, there is no free lunch!

Then, for those situations in which the Bayesian approach is not broken, two questions arise naturally:

1. When does a proper scoring rule model comparison produce a different answer than a log-score model comparison?
2. For those situations in which the answers are different, can an argument be made for preferring the proper scoring rule method?

This suggests juxtaposing the proposed method to model comparison methods that compare directly the (log-) likelihoods for the various models.

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