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REJOINDER TO DISCUSSIONS OF "FREQUENTIST COVERAGE OF ADAPTIVE NONPARAMETRIC BAYESIAN CREDIBLE SETS"

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We thank the discussants for their supportive comments and interesting observations. Many questions are still open and not all methodological or philosophical questions may have an answer. Our reply addresses only a subset of questions and is organized by topic. A final section reviews recent work.

1. Hierarchical Bayes credible sets. Our paper considers empirical Bayes tuning of the posterior distribution, whereas many Bayesians might prefer to use a hierarchical Bayes approach. Ghosal and Rousseau ask whether, or conjecture that, the hierarchical Bayes procedure behaves similarly as the likelihood based empirical Bayes procedure. Indeed, we can show exactly the same coverage of hierarchical Bayes credible sets for polished tail truths. A counterexample showing that hierarchical Bayes credible sets also do *not* cover without some restriction was already given in [14], while the size of such sets follows from [7]. Thus, within the context of our paper there is no difference between the two schemes.

In the hierarchical Bayes approach we endow the regularity hyperparameter α with a hyperprior distribution λ, and then apply an ordinary Bayes method with the overall prior, for some upper bound A (possibly dependent on n),

$$\Pi(\cdot) = \int_0^A \Pi_\alpha(\cdot)\lambda(\alpha)\, d\alpha.$$

For $\Pi(\cdot|X^{(n)})$ the posterior distribution relative to this prior, a hierarchical Bayes credible ball centered at the posterior mean $\hat{\theta}_n$ is defined by its radius $\hat{r}_{n,\gamma}$:

$$\Pi\bigl(\theta : \|\theta - \hat{\theta}_n\|_2 \le \hat{r}_{n,\gamma} | X^{(n)}\bigr) = 1 - \gamma. \tag{1.1}$$

We blow this up a bit, and for $L > 0$ consider

$$\hat{C}_n(L) = \bigl\{\theta \in \ell^2 : \|\theta - \hat{\theta}_n\|_2 \le L\hat{r}_{n,\gamma}\bigr\}. \tag{1.2}$$

Under a mild regularity condition on λ, similar to that in [7], these sets cover polish tail truths.

THEOREM 1.1. *Suppose that there exist $c_1, c_2 \ge 0$, c_3 and $c_4, c_5 > 0$, with $c_3 > 1$ if $c_2 = 0$, such that $c_4^{-1}\alpha^{-c_3}\exp(-c_2\alpha) \le \lambda(\alpha) \le c_4\alpha^{-c_3}\exp(-c_2\alpha)$, for*

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