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DISCUSSION OF "FREQUENTIST COVERAGE OF ADAPTIVE NONPARAMETRIC BAYESIAN CREDIBLE SETS"[1]

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We congratulate the authors for this very interesting article focused on the frequentist coverage of Bayesian credible sets in the context of an infinite-dimensional signal in white noise models. In such settings the construction of honest confidence sets is especially complicated, at least when the goal is to construct confidence sets that have a size that adapts to the unknown parameters in the model, while maintaining coverage probability.

The focus of the present paper is on constructing l_2 balls as confidence sets. There are some advantages that come with the focus on balls for confidence sets. For bands results in Low (1997) rule out the possibility of adaptation over even a pair of Lipschitz or Sobolev spaces at least for confidence bands that have a guaranteed coverage level. On the other hand, fully rate adaptive confidence balls which do maintain coverage probability can be constructed over Sobolev smoothness levels that range over an interval $[\alpha, 2\alpha]$. However, this range of models where such adaptation is possible is still quite limited and here the authors develop a theory that applies over a broader class of models. The approach taken, following Giné and Nickl (2010) and Bull (2012), is to focus on parameters that are in some sense typical and removing a set of parameter values that cause difficulties at least when constructing adaptive sets. The goal is then to construct fully adaptive confidence sets over the remaining collection of parameter values. In the present paper the parameter values that are kept belong to a class of parameters that they call polished tail sequences and the authors develop results that show that a particular empirical Bayes credible ball is both honest when restricted to such sequences and adaptive in size.

There are of course many settings where it is more natural to focus on the construction of confidence bands rather than confidence balls and, typically, theory and methodology developed for balls do not provide a way to also construct bands. Here, however, the balls are constructed from an empirical Bayes posterior and even though the focus of the paper is on the construction of balls, the simulation example in Section 4 suggests that a general methodology for the construction of confidence bands can also be developed based on this posterior. The visualization of the credible sets is constructed by making draws from the empirical Bayes posterior and plotting the 95% that are closest in l_2 to the posterior mean. Each draw gives rise to an entire function, but visually the appearance is somewhat akin to a

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