

Rejoinder: Analysis of AneuRisk65 data*

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We would like to thank the various groups who analyzed the AneuRisk65 data, providing a broad range of analyses with many interesting results. This dataset offers the possibility of performing registration and analyses at various data levels. Cheng et al. (2014) and Xie, Kurtek and Srivastava (2014) registered the three dimensional vessel centrelines, likewise in Sangalli et al. (2009, 2010). Staicu and Lu (2014) and Gervini (2014) opted instead for the corresponding one-dimensional curvature functions. The registration methods used are varied, some (see Cheng et al., 2014; Sangalli, Secchi and Vantini, 2014) only allowing for shift or linear warpings of the abscissa parameters, others (see Staicu and Lu, 2014; Xie, Kurtek and Srivastava, 2014; Gervini, 2014) allowing for very flexible warpings. One difficulty with these data is that the portion of reconstructed vessel morphology has different length across subjects: for some subjects, only the terminal tract of the Inner Carotid Artery (ICA) is observed and reconstructed, for other patients a long portion of the arterial tract preceding the distal part is also available. The observed vessel lengths are significantly different for the considered groups of subjects – Upper and Lower-No groups – due to the different location of the scanned volume. Some registration methods (see Cheng et al., 2014; Sangalli, Secchi and Vantini, 2014; Gervini, 2014) explicitly tackle this issue, whilst others (see Staicu and Lu, 2014; Xie, Kurtek and Srivastava, 2014) map all curves to a common domain. On the other hand, we believe that the more flexible warping functions considered by these latter methods might have counterbalanced the problem, at least to some extent, by re-stretching toward the terminal part shorted arteries and by unwinding longer ones. After registration, most groups (see Cheng et al., 2014; Staicu and Lu, 2014; Gervini, 2014) try to discriminate Upper and Lower-No groups analyzing the vessel shape or some morphological feature such as vessel radius and curvature, similarly to Sangalli et al. (2009). As expected, when a rich family of warping functions is used, the discrimination improves if the phase variability captured by the warping functions is also explicitly considered in the discrimination method. Xie, Kurtek and Srivastava (2014) instead perform an unsupervised clustering of the vessels based on their three-dimensional shapes, analogously to Sangalli et al. (2010).

Here are some comments specific to each data analysis.

Cheng et al. (2014) analyze the data using techniques from shape analysis. The vessel centrelines are registered in the three-dimensional space by

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