

## Rejoinder

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A good deal of thanks go out to all of the discussants for their insightful and interesting comments, to the referees for their help in improving the paper, and to the editors for enabling this discourse.

Broadly speaking, the criticisms and suggestions of the discussants pointed to several theoretical and applied weaknesses. Without respect to any specific modeling goal, the Uhlig extension as originally presented suffers from several unappealing features: it uses only a few fixed parameters to model large, time-varying quantities, which is a bit too parsimonious, and the evolution of the hidden states is problematic. When examining the model within the context of finance, the model misses many key features and disagreements between observed statistical regularities and those captured by the model are brought into relief.

Thankfully, the discussants not only identified many shortcomings, but also provided many solutions.

We very much appreciate the proposed improvements in the discussions of both Forbes and Casarin. Within the context of a financial time series, Forbes has shown how to elegantly incorporate jumps into the dynamics of the price process while preserving all of the machinery of the Uhlig extension; Casarin has shown not only why one should want to use time-varying parameters, but how to incorporate them via a Markov-switching approach.

Regarding Casarin's suggested improvements (see Section 3), one must be careful when letting the degrees of freedom parameters change. He suggests

$$\begin{cases} \mathbf{Y}_t \sim W_m(k_t, (k_t \mathbf{X}_t)^{-1}), \\ \mathbf{X}_t \sim \mathbf{T}'_{t-1} \boldsymbol{\Psi}_t \mathbf{T}_{t-1} / \lambda, & \boldsymbol{\Psi}_t \sim \beta_m \left( \frac{n_t}{2}, \frac{k_t}{2} \right), \\ \mathbf{T}_{t-1} = \text{upper chol } \mathbf{X}_{t-1} \end{cases}$$

where  $m$  is the order of the matrices involved. Given the initial distribution  $(\mathbf{X}_0 | \mathcal{D}_0) \sim W_m(n_0 + k_0, (k_0 \boldsymbol{\Sigma}_0)^{-1})$ , the filtered and predictive distributions evolve in the following way (we continue to use the notation  $\mathcal{D}_t = (\mathbf{Y}_1, \dots, \mathbf{Y}_t)$  and we implicitly condition on

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