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Rejoinder

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We are very grateful to all discussants of this paper for their thought–provoking and constructive comments and the interesting research directions they indicate. We present our reply to the comments of the discussants in alphabetical order.

[**J. M. Bernardo**] We agree on the importance of checking the asymptotic normality of the joint posterior distribution induced by the Jeffreys prior, provided it exists. However, we have shown that, in the context of two–piece location–scale models, the Jeffreys–rule prior does not even lead to a proper posterior distribution when the underlying symmetric density f belongs to the family of scale mixture of normals. This result precludes the use of the Jeffreys–rule prior for conducting Bayesian inference altogether.

Although we do not claim optimality of the Jeffreys–rule nor the independence Jeffreys priors in any sense, the optimality of a prior depends, of course, on the optimality criterion. While the Jeffreys–rule prior is not optimal for inferences in certain cases, it represents the optimal choice in some others (see *e.g.* Hedayati and Barlett 2012). Therefore, we think the study of these priors is of interest, especially if this study reveals important pitfalls that occur when such priors are used in apparently simple models. The reality is that Jeffreys priors offer a relatively easily obtained prior candidate in the absence of strong prior information, which has some interesting properties, such as invariance with respect to reparameterizations, and is used quite a lot in practice. In fact, a quick search on Google Scholar (on January 20, 2014) reveals 423 papers since 2013 with the term "Jeffreys prior" (about the same number as containing the phrase "reference prior").

Given the impropriety of the posterior induced by the Jeffreys–rule prior, we present some alternative priors that lead to proper posteriors under certain conditions. The first (standard) alternative studied is the independence Jeffreys prior, which often performs better in practical situations. The Jeffreys–rule and the independence Jeffreys prior coincide for orthogonal parameterizations, which in this case do not seem to be achievable, as discussed by Jones and Anaya-Izquierdo (2011). The second class of alternative priors consists of modifications of the Jeffreys–rule and the independence Jeffreys priors on a more heuristic basis. These priors lead to proper posteriors and, as shown in the Supplementary material (Appendix 2), in a simulation study they also result in posteriors with good frequentist properties.

The study of other types of noninformative priors represents an interesting extension of our work, and the reference prior would indeed be the first alternative we would try.

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