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DISCUSSION OF "FREQUENTIST COVERAGE OF ADAPTIVE NONPARAMETRIC BAYESIAN CREDIBLE SETS"

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1. Introduction. I would like to congratulate Botond Szabó, Aad van der Vaart and Harry van Zanten [12] for a fundamental and thought provoking article on a highly important topic. One of the key contributions of statistics to modern science may arguably be *the theory of uncertainty quantification*. Assessing the accuracy of an estimate by a confidence statement goes beyond the mere search for an efficient statistical algorithm. In particular, within the contemporary search for *adaptive* procedures, research of the last decade has revealed that the construction of adaptive confidence statements is fundamentally *harder*—in an information theoretic sense—than the construction of adaptive algorithms. Confidence statements are at the same time crucial for the main application of modern data analysis, which is to accept or reject hypotheses.

Szabó, van der Vaart and van Zanten tackle the important topic as to whether increasingly popular Bayesian methodology can actually provide objective uncertainty quantification methods in nonparametric models or not. The nonparametric setting is a key test-case for the general paradigm of high-dimensional modeling that has emerged recently in statistics.

My discussion of the paper surrounds the two focal points of why "Bayesian uncertainty quantification" is a mathematically and conceptually nontrivial problem: the first has nothing to do with adaptation and addresses some of the probabilistic subtleties intrinsic to the Bayesian approach to provide "credible sets." The second point is common to all frequentist procedures and is about the fact that adaptive uncertainty quantification is in general only possible under "signal-strength" conditions on the underlying parameter.

2. Freedman's paradox and the nonparametric Bernstein–von Mises theorem. I first want to discuss the fact that the frequentist coverage probabilities obtained by Szabó, van der Vaart and van Zanten for their credible sets are not *exact*, that is to say, not of the precise asymptotic level $1-\alpha$, and the related question of why obtaining exact posterior asymptotics in the nonparametric situation is a subtle matter.

Consider observations $Y \sim P_\theta$ with parameter space $\theta \in \Theta$, a prior Π on Θ and resulting posterior distribution $\Pi(\cdot|Y)$ of $\theta|Y$. The classical finite-dimensional

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