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DISCUSSION OF "ESTIMATING THE HISTORICAL AND FUTURE PROBABILITIES OF LARGE TERRORIST EVENTS" BY AARON CLAUSET AND RYAN WOODARD

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We congratulate the authors on this well-written and thought-provoking paper. They address the problem of estimating the probability of a large (and rare) terrorist attack by modeling the tail of the attack size distribution. Recognizing the importance of incorporating uncertainty, their approach uses bootstrap resampling to obtain a set of parameter estimates for the tail distribution from which estimates for the probability of the rare event can be made. The wide range for the estimated probability of a 9/11-sized attack (90% interval [0.182, 0.669]) illustrates the need to account for uncertainty in such a problem.

The authors also recognize that the choice of tail model can have a large impact on the probability estimates. Using multiple tail models (power law, stretched exponential and log-normal), they estimate that the probability of a 9/11-sized attack over a 40-year period (or, more specifically, in 13,274 events) ranges from around 11–35%. We thought it would be interesting to compare the results of the authors' analysis with a more classical extreme value analysis [Coles (2001), de Haan and Ferreira (2006)] using a generalized Pareto distribution (GPD). The GPD distribution has three parameters: lower bound μ, scale σ and shape ξ. If $Y \sim \mathrm{GPD}(\mu, \sigma, \xi)$, then Y's cumulative density function is

$$F(y|\mu,\sigma,\xi) = 1 - \left(1 + \frac{\xi(y-\mu)}{\sigma}\right)^{-1/\xi}. \tag{1}$$

The shape parameter ξ determines the support of Y. If $\xi < 0$, then Y is bounded to the interval $\mu < Y < \mu - \sigma/\xi$; if $\xi > 0$, then Y is unbounded with support $Y > \mu$. The shape parameter also determines the tail behavior. If $\xi < 0.5$, then the density has light tails and finite mean and variance. Large ξ gives heavy tails. If $\xi > 0.5$, the variance in infinite, and if $\xi > 1$, then the mean is also infinite. If $\xi > 0$ and $\sigma = \mu \cdot \xi$, then the GPD reduces to the (continuous) power-law distribution.

Asymptotic theory suggests that the GPD provides a good approximation for the tail of a wide range of densities. The typical approach is to select a lower bound μ based on exploratory analysis, discard the data below μ, and estimate σ and ξ using maximum likelihood. A crucial step in this analysis is to select an

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