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Discussion of "Statistical Modeling of Spatial Extremes" by A. C. Davison, S. A. Padoan and M. Ribatet

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We congratulate the authors for their overview paper discussing modeling techniques for spatial extremes. There is great interest in spatial extreme data in the atmospheric science community, as the data is inherently spatial and it is recognized that extreme weather events often have the largest economic and human impacts. In order to adequately assess the risk of potential future extreme events, there is a need to know how the characteristics of phenomena such as precipitation or temperature could be altered due to climate change.

Because of the high interest level in the atmospheric science and (more broadly) the geoscience communities, it is imperative for the statistics community to develop methodologies which appropriately answer the questions associated with spatial extreme data. Davison, Padoan and Ribatet (2012) provide a comprehensive overview of existing techniques that can serve as a useful starting point for statisticians entering the field. That the paper is written as a case study helps to illustrate the advantages and disadvantages of the various methods. We hope that this Swiss rainfall data will serve as a test set by which future methodologies can be evaluated.

The authors analyze data which are annual maxima. This is natural from the classical extreme value theory point of view whose fundamental result establishes the limiting distribution of $\mathbf{Y} = (\bigvee_{i=1}^{n} X_{1i}, \ldots, \bigvee_{i=1}^{n} X_{Di})^T$ to be in the family of the multivariate max-stable distributions. In practice, modeling vectors of annual maxima seems less than ideal, and it is not clear how much dependence information is lost by discarding the coincident data. Scientists in other disciplines can be uncomfortable with the idea of constructing data vectors of events which most often occur on different days. We are aware that there is current work to extend spatial extremes work to deal with threshold exceedances, and we look forward to that work appearing in the literature.

Davison, Padoan and Ribatet (2012) divide the spatial approaches into three categories: latent variable models, copulas, and max-stable process models. In Section 7 they do a very nice job of detailing the strengths and weaknesses of the three approaches. However, it seems that the article does not make clear enough that the aim of the latent variable approach is fundamentally different than the aim of a copula or max-stable process model. As the authors state in Sections 2.2 and 2.3, current modeling of multivariate (or spatial) extremes requires two tasks: (1) the marginals must be estimated and transformed to something standard (e.g., unit Fréchet) so that (2) the tail dependence in the data can be modeled. The latent variable model is a method for characterizing how the marginal distribution varies over space, that is, task 1. In contrast, both copula models and existing max-stable process models explicitly model the tail dependence in the data once the marginals are known, that is, task 2. We refer to the dependence remaining after the marginals have been accounted for as "residual dependence," as Sang and Gelfand (2010) described the random variables after marginal transformation as "standardized residuals."

Davison, Padoan and Ribatet (2012) are correct to point out (Figure 4) that using a latent variable model is inappropriate for applications where the joint behavior of the random vector is required. However, there are applications which aim only to model the marginal behavior. There is a long history of producing return level maps such as those shown in Figure 3 of the manuscript. For instance, the recent effort to update the precipitation frequency atlases for the US (Bonnin et al., 2004a, 2004b) aimed only to characterize the marginal distribution's tail over the study region. Bonnin et al. (2004a, 2004b) employed a regional frequency analysis (Dalrymple, 1960; Hosking and

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