Bayesian Analysis (2012) **7**, Number 2, pp. 273–276

Rejoinder

Kristian Lum* and Alan E. Gelfand†

We very much appreciate these three diverse discussions with virtually no overlap across them. We first take up the comments of Guhaniyogi and Banerjee (henceforth GB). With regard to the association structure under the asymmetric Laplace process (ALP), perhaps our presentation was not as clear as it should have been. Working with say isotropic covariance functions, we find that, regardless of the specification for the $\xi(s)$, the resulting correlation depends only on the distance between locations and is symmetric in p away from .5. Explicitly,

$$\text{corr}(\epsilon(s), \epsilon(s')) = \frac{\rho(||s - s'||)E(\sqrt{U(s)U(s')} + b_p \text{corr}(U(s), U(s')))}{1 + b_p}$$

where marginally, the $U(s) \sim \text{Exp}(1)$ and $b_p = \frac{(1-2p)^2}{2p(1-p)}$. Note that b_p is minimized at 0 when $p = .5$ and tends to ∞ as $p \to 0, 1$. With a common $U(s) = U$, we see that regardless of s and s', the correlation can not go to 0, taking its minimum at $p = .5$, tending to 1 as $p \to 0, 1$. We don't employ this case. With a copula spatial process model for $\xi(s)$, equivalently, $U(s)$, we find that, for any p, correlation will go to 0 as $||s - s'|| \to \infty$. Again, it will take its minimum at $p = .5$, tending to $\text{corr}(U(s), U(s'))$ as $p \to 0, 1$, given s and s'. We don't employ this case either. For the case of i.i.d. $\xi(s)$, the second term in the numerator disappears and the expectation in the first term is constant ($\pi/4$). So now, for any p, correlation will go to 0, as determined by ρ and is strongest at $p = .5$, tending to 0 regardless of s and s' as $p \to 0, 1$. This behavior seems to be what would be desired for the $\epsilon(s)$ process.

With regard to the asymmetric Laplace predictive process (ALPP), we liked the novel form of the "bias" adjustment that arises due to the constraint that $\text{var}\ddot{Z}(s)$ must be 1. The tapered adjustment form in Sang and Huang (2012) is attractive but, we agree that its use is not likely to affect the inference in the present context. We concede that employing the double Gaussian process, drawing from Kottas and Krnjajić (2009) would be more flexible than the ALP and is investigated in the thesis of Lum (2010). Its properties and implementation issues are discussed there but presentation was beyond the scope of this paper. Finally, we do like the GB idea of joint modeling of spatial quantiles, imagining an application for modeling quantiles of ozone and $PM_{2.5}$ exposure.

We must take issue with the discussion of Lin and Chang (henceforth LC). They present a simulation example which seemingly reveals some shortcomings of the ALP. They claim that because our method does not perform well under the loss function they suggest, $SSE(p) = (q_p - \hat{q}_p)^2$, it does not provide the same flexibility as its semi-parametric frequentist cousin.

First, we disagree that we are using a mean regression model. We are certainly not

*Unaffiliated, kristianlum@gmail.com
†Department of Statistical Science, Duke University, Durham NC, alan@stat.duke.edu

 DOI:10.1214/12-BA708REJ