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Comment on Article by Lum and Gelfand

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We congratulate the authors for a well-written article on a problem of clear and increasing scientific relevance. Quantile regression is being widely deployed in a number of disciplines to harness additional information about the relationship between the outcome and covariates at the extremes of the outcome's distribution. Modeling this in the context of spatially referenced data is challenging. We have a few comments on some aspects of the model.

The authors have argued the importance of conditional spatial quantile models to account for varying effects of covariates across quantiles. To model these conditional spatial quantiles of the response, they deploy the asymmetric Laplace process (ALP). An elegant characterization of this distribution in terms of Gaussian and Gamma random variables constitutes the premise of constructing spatial ALP's. A particularly attractive feature of this approach is the ease with which the Markov chain Monte Carlo (MCMC) samplers can be designed, not only to update model parameters but also to carry out spatial interpolation at arbitrary locations.

A point worth noting is that the ALP process can induce high correlations between two outcomes at extreme quantiles ($p \to 0$ and $p \to 1$), even if the corresponding locations are distant. In other words, outcomes arising from the tails of the distribution are assumed to be highly correlated, irrespective of how far away in space they have been collected. This is perhaps why the authors restricted their inference to quantiles between 0.2 and 0.8 in the Baton Rouge real estate data example. We wonder how serious this issue is in practice and whether their methodology is rendered invalid for applications desiring more accurate inference on higher quantiles (see Reich et al. 2011).

We, however, recognize the flexibility of the ALP process to adapt to a more generalized set up. In particular, the Gaussian-Gamma representation can be easily extended to arrive at multivariate asymmetric Laplace processes (see Kotz et al. 2001, references therein). The multivariate ALP is characterized by a multivariate normal random variable and a gamma random variable. Analogous to the univariate setting, the multivariate normal random variable can be replaced by a multivariate Gaussian process. It is not hard to envision rich multivariate spatial quantile regression models for settings where interest lies in jointly modeling a quantile for multiple spatially referenced outcomes. A natural choice for the error process is the multivariate ALP. Inference will then boil down to modeling the matrix-valued cross-covariance function of the multivariate Gaussian process.

We also appreciate the attention to large spatial datasets for which the authors have devised the asymmetric Laplace predictive process (ALPP). The Gaussian predictive process, $\tilde{w}_m(\boldsymbol{s})$ in Section 7.1, emerges as a conditional expectation of the full-rank

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