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CORRECTION ON MOMENTS OF MINORS OF WISHART MATRICES

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Theorem 5.7 in [1] gives a formula for the variance of a minor (i.e., a subdeterminant) of a Wishart random matrix. The formula has to be corrected as follows.

THEOREM 5.7. *Let $I, J \in \binom{r}{m}$ have intersection $C := I \cap J$ of cardinality $c = |C| = |I \cap J|$. Define $\bar{I} = I \setminus (I \cap J)$, $\bar{J} = J \setminus (I \cap J)$ and $\bar{I}\bar{J} = \bar{I} \cup \bar{J}$. Then the minor* $\det(S_{I\times J}) = \det(S_{IJ})$ *of the Wishart matrix $S \sim \mathcal{W}_r(n, \Sigma)$ has variance*

$$
\begin{aligned}
&\mathrm{Var}[\det(S_{IJ})]\\
&\quad = \det(\Sigma_{IJ})^2 \frac{n!}{(n-m)!}\left[\frac{(n+2)!}{(n+2-m)!} - \frac{n!}{(n-m)!}\right]\\
&\qquad + \det(\Sigma_{C\times C})^2 \det(\bar{\Sigma}_{\bar{I}\bar{J}\times\bar{I}\bar{J}}) \frac{(n+2)!}{(n+2-c)!}\cdot\frac{n!}{(n-m)!}\\
&\qquad\quad \times \left[\sum_{k=0}^{m-c-1}(m-c-k)!\frac{(n+2-c)!}{(n+2-c-k)!}(-1)^k \operatorname{tr}\{(\bar{\Sigma}_{\bar{I}\bar{J}}\bar{\Sigma}^{\bar{I}\bar{J}})^{(k)}\}\right],
\end{aligned}
$$

where $\bar{\Sigma} = \Sigma_{([r]\setminus C)\times([r]\setminus C)} - \Sigma_{([r]\setminus C)\times C}\Sigma_{C\times C}^{-1}\Sigma_{C\times([r]\setminus C)}$.

PROOF. Define $\bar{S}$ in analogy to $\bar{\Sigma}$. Since $\det(S_{IJ}) = \det(S_{C\times C})\det(\bar{S}_{\bar{I}\times\bar{J}})$, and $S_{C\times C}$ and $\bar{S}_{\bar{I}\times\bar{J}}$ are independent (Lemma 5.2),

$$
\begin{aligned}
\mathrm{Var}[\det(S_{IJ})] = {}&\big(\mathrm{Var}[\det(S_{CC})] + \mathrm{E}[\det(S_{CC})]^2\big)\,\mathrm{Var}[\det(\bar{S}_{\bar{I}\bar{J}})]\\
&+ \mathrm{Var}[\det(S_{CC})]\mathrm{E}[\det(\bar{S}_{\bar{I}\bar{J}})]^2.
\end{aligned}
$$

The claim now follows from Corollary 4.2 and Propositions 5.1 and 5.5. □

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