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Rejoinder

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1. INTRODUCTION

Very sincere thanks to the discussants for choosing to enter a virtual minefield of disagreement in the development history of statistics. For we just need to recall the remark that fiducial is Fisher's "biggest blunder" and place it alongside the fact that fiducial was the initial step toward confidence, which arguably is the most substantive ingredient in modern model-based theory: the two differ in minor developmental detail, with fiducial offering a probability distribution as does Bayes and with confidence offering just probabilities for intervals and special regions. Statistics has spent far more time attacking incremental steps than it has seeking insightful resolutions.

As a modern discipline statistics has inherited two prominent approaches to the analysis of models with data; of course such is not all of statistics but is a critical portion that influences the discipline widely. How can a discipline, central to science and to critical thinking, have two methodologies, two logics, two approaches that frequently give substantially different answers to the same problems. Any astute person from outside would say, "Why don't they put their house in order?" And any serious mathematician would surely ask how you could use a lemma with one premise missing by making up an ingredient and thinking that the conclusions of the lemma were still available. Of course, the two approaches have been around since 1763 and 1930 with regular disagreement and yet no sense of urgency to clarify the conflicts. And now even a tired discipline can just ask, "Who wants to face those old questions?": a fully understandable reaction! But is complacency in the face of contradiction acceptable for a central discipline of science?

A statistical model differs from a deterministic model in having added probability structure that describes the variability typically present in most applications. So, in an application with a statistical model and related data it would then seem quite natural that that variability would enter the conclusions concerning the unknowns in an application: what do I know deterministically, and what do I know probabilistically?

And that is what Bayes proposed in 1763: probability statements concerning the unknowns of an investigation. Many have had doubts and said there was no merit in the proposal; and many have acceded and became strong believers. And then Fisher (1930) also offered probabilities concerning the unknowns of an investigation, but by a different argument, and the turf fight began! Bayes had hesitantly examined a special problem and *added* a random generator for the unknown parameter, and Fisher had worked more generally and used just the randomness that had generated the data itself.

But then a third person, Lindley (1958), from the same country said that the second person, Fisher, couldn't use the term probability for the unknowns in an investigation, as the term was already taken by the first person, Bayes. And strangely the discipline complied! Decades went by and anecdotes were traded and things were often vitriolic.

2. WHAT DOES THE ORACLE SAY?

Consider some regular statistical model $f(y;\theta)$, together with a lower β-confidence bound $\hat{\theta}_\beta(y)$, and also a lower β-posterior bound $\tilde{\theta}(y)$ based on a prior $\pi(\theta)$: What does the oracle see concerning the usage of these bounds? He can investigate any long sequence of usages of the model, and He would have available the data values y_i and of course the preceding parameter values θ_i that produced the y_i values; He would thus have access to $\{(\theta_i, y_i): i = 1, 2, \ldots\}$.

First consider the lower confidence bound. The oracle knows whether or not the θ_i is in the confidence interval $(\hat{\theta}_\beta(y_i), \infty)$, and He can examine the long-run proportion of true statements among the assertions that θ_i is in the confidence interval $(\hat{\theta}_\beta(y_i), \infty)$, and He can see whether the confidence claim of a β-proportion true is correct. In agreement with the mathematics of confidence, that proportion is just β.

Now consider the lower posterior bound. The oracle knows whether θ_i is in the posterior interval $(\tilde{\theta}_\beta(y_i), \infty)$, and He can examine the long-run proportion of true statements that θ_i is in the posterior

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