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Rejoinder

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We would like to thank all the discussants for their wide-ranging comments. Before we respond to them individually in alphabetical order, we would like to address some general issues first. As we have said, we have chosen to describe our aim of matching the joint distribution of the observable data as *feature matching*, for want of a better name. We should have perhaps emphasized that we regard cycles, spectral singularities, and so on only as partial aspects of the joint distribution. They are useful, in practical applications, only in so far as they can provide partial measures of feature matching. We think Professor Hansen has understood our aim well in his introduction. We have sometimes, for brevity, called our general approach to achieving this aim the *catch-all approach*. We should stress the following point once more. The catch-all approach is not restricted to catch all first-order (conditional) moments or catch all second-order moments. We have used them in the paper primarily as illustrations of what the approach can deliver in modeling, beyond conventional methods based on the one-step-ahead prediction errors. Clearly, once the catch-all idea is accepted, we can equally well catch all kth-order (conditional or unconditional) moments, catch all marginal (conditional or unconditional) distributions, and so on. Moreover, the objective function Q can also take on a form other than that of a mean squared type; for example, it can be of a likelihood type as stated in Section 2.1.

Professors Chan and Tsay have tried the catch-all approach on two real data sets, namely (i) the CREF stock fund and (ii) the monthly global temperature anomalies from 1880 to 2010. In each case, their implementation of the approach is exemplary. In data set (i), the catch-all approach has led to parameter estimates of the postulated GARCH(1, 1) model that enable the model to "track the squared returns more closely" and "transit into the ensuing quiet period at a faster rate commensurate with the data." We are sure that Chan and Tsay are aware of the fact that the larger is α, the more responsive is the GARCH(1, 1) model to volatility.

Chan and Tsay seem to be disappointed with their attempt with data set (ii). They have correctly noted the shapes of the eventual forecasting functions (eff) of the ARIMA(1, 1, 1) model and the ARMA(1, 1)-plus-trend model. Now, long-range forecasting invokes a low pass filter, which is approximately provided by the eff. Therefore, for an ARIMA(1, 1, 1) model, for sufficiently large l and conditional on $Y_s, s \le t$, $EY_{t+l} \approx K$, where K is a constant. In such cases, $\phi \approx -\theta$, the well-known near cancelation of the AR operator and the MA operator. Similar arguments apply to an ARMA(1, 1) model. It is clear in the setup of Chan and Tsay, as m increases, long-range forecasts exert greater and ultimately overwhelming influence on the objective function, S. Thus, evidence of operator near cancelation with increasing m is evidence of plausibility of the postulated model. This argument suggests that if Chan and Tsay had perhaps probed further with their Figure 2, they might be marginally more inclined toward the ARMA(1, 1)-plus-trend model. Of course, we must always be very cautious if we entertain any thought of extrapolating the trend into the future.

Taking up the challenge posed by Chan and Tsay relating to business cycles, we have considered the unemployment rate in the United States. The second panel of Figure 1 shows the rate after the removal of a moving mean. The partial autocorrelation function suggests strong AR(2) effect with a hint of higher order dependence. Figure 1 compares the spectral density functions of the AR model, from order 2 to 5, fitted respectively by the catch-all approach and the maximum likelihood approach. The former approach seems to show an overall better matching of the observed. The fundamental period of 9 years is clearly discernible and reasonably well captured by the AR(3), AR(4) and AR(5) models fitted by the catch-all approach. The possible existence of higher harmonics deserves further investigation, however.

Professor Hansen has made numerous perceptive comments. We are much heartened by his endorsement

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