Statistical Science
2011, Vol. 26, No. 1, 53–56
DOI: 10.1214/11-STS345B
Main article DOI: 10.1214/10-STS345

Discussion of “Feature Matching in Time Series Modeling” by Y. Xia and H. Tong

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We thank Xia and Tong for their stimulating article on time series modeling. Their emphasis on estimation rather than model specification is interesting. It brings new light to statistical applications in general and to time series analysis in particular. The use of maximum likelihood or least squares method is so common, especially with the widely available statistical software packages, that one tends to overlook its limitations and shortcomings.

There is hardly any statistical method or procedure that is truly “one-size-fits-all” in real applications. We welcome Xia and Tong’s contributions as they argue so convincingly that feature matching often fares better in time series modeling. On the other hand, we’d like to point out some issues that deserve a careful study.

1. HIGHER ORDER PROPERTIES

The conditional mean function generally provides a good description of the cyclical behavior of the underlying process, and the catch-all approach can be effectively implemented by estimating the model that matches the multi-step conditional means to the data, as eminently illustrated by the authors. Here, we want to point out the natural extension of estimating a model by matching multi-step conditional higher moments to the data. For example, in financial time series analysis, it is pertinent to model the dynamics of the conditional variances. Consider the simple case that a time series of returns, $\{r_t\}$, follows a generalized autoregressive conditional heteroscedastic model of order $(1, 1)$ or simply a GARCH$(1, 1)$ model:

$$\begin{aligned} r_t &= \sigma_{t|t-1}\varepsilon_t, \\ \sigma^2_{t|t-1} &= \omega + \alpha r^2_{t-1} + \beta\sigma^2_{t-1|t-2}, \end{aligned}$$

where $\omega > 0, \alpha \ge 0, \beta \ge 0$, $1 > \alpha + \beta > 0$ are parameters, $\{\varepsilon_t\}$ are independent and identically distributed (i.i.d.) random variables with zero mean and unit variance, and ε_t is independent of past one-step-ahead conditional variances $\sigma^2_{s|s-1}, s \le t$. Estimation of the GARCH model can be done by maximizing the Gaussian likelihood of the data, which essentially matches the conditional variances with the squared returns.

A natural generalization of the catch-all method is to estimate a GARCH model that matches the k-step-ahead conditional variance to the kth ahead data, for $k = 1, 2, \ldots, m$ with a fixed m, by minimizing some weighted measure of dissimilarity of the multi-step conditional variances to future squared returns. Various dissimilarity measures may be used. Here, we illustrate the usefulness of this idea by adopting the negative twice Gaussian log-likelihood as the dissimilarity measure, that is, estimating a GARCH model by minimizing

$$S(\omega, \alpha, \beta) = \sum_{t=1}^{n-m}\sum_{\ell=1}^{m} w_\ell\{r^2_{t+\ell}/\sigma^2_{t+\ell|t} + \log(\sigma^2_{t+\ell|t})\},$$

where $\{w_\ell\}$ is a set of fixed weights and $\sigma^2_{t+\ell|t}$ is the conditional variance of $r_{t+\ell}$ given information available at time t. When $m = 1$, the new method reduces to the Gaussian likelihood method. On the other hand, under the assumption that the true model is a GARCH model and for a fixed $m > 1$, the estimator is expected to be consistent and asymptotically normal, with details of the investigation to be reported elsewhere. However, if the GARCH model does not contain the true model, as likely is the case in practice, the (generalized) catch-all method with $m > 1$ may provide new information for estimating a GARCH model that better matches the observed volatility clustering pattern.

We tried this approach by fitting a GARCH$(1, 1)$ model to the daily returns of a unit of the CREF stock fund over the period from August 26, 2004 to August 15, 2006; this series was analyzed by Cryer and Chan [(2008), Chapter 12], and they identified the series as a GARCH$(1, 1)$ process. Gaussian likelihood

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